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JEE (Main)

PAPER-2 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 29 June, 2022 (SHIFT-2) | TIME : (3.00 p.m. to 06.00 p.m)
Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. The value of $\lim\limits_{x\to 1}\dfrac{(x^2-1)\sin^2\pi x}{x^4-2x^3+2x-1}$ is

(1) π (2) π^2 (3) 2π (4) $\dfrac{\pi}{2}$

Ans. **(2)**
Sol. Given

$$\lim_{x\to1}\frac{(x^2-1)\sin^2\pi x}{x^4-2x^3+2x-1}$$

$$= \lim_{x\to1}\frac{(x-1)(x+1)\sin^2\pi x}{(x-1)^3(x+1)}$$

Put x = 1 + h

$$= \lim_{h\to0}\frac{\sin^2\pi(1+h)}{(1+h-1)^2}$$

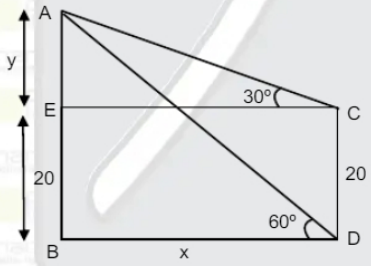
$$\lim_{h\to0}\pi^2\frac{\sin^2\pi h}{\pi^2h^2} = \pi^2$$

2. The angle of elevation of the top of a tower from the top and bottom of a pole are 30° and 60° respectively. If the height of pole is 20m then the height of the tower is (in meters)

(1) 35 (2) 30 (3) 40 (4) 45

Ans. **(2)**

Sol.



In ΔABD

$$\tan60^\circ = \frac{20+y}{x} \quad \text{.....(1)}$$

In ΔAEC

$$\tan30^\circ = \frac{y}{x} \quad \text{.....(2)}$$

from equation $\frac{\tan60^\circ}{\tan30^\circ} = \frac{20+y}{y} \Rightarrow \frac{\sqrt{3}}{\sqrt{1/3}} = \frac{20+y}{y}$

$$\Rightarrow \frac{20+y}{y} = 3 \Rightarrow y = 10$$

height of tower = 20 + y = 30m

3. Negation of $(p\wedge q) \Rightarrow (\sim p)\wedge r$ is

(1) $p\vee q$ (2) $p\wedge q$ (3) $(p\wedge q)\vee\sim r$ (4) $p\vee\sim r$

Ans. **(2)**

Sol. Given $(p\wedge q) \Rightarrow (\sim p)\wedge r$

$\therefore$ Negation is $(p\wedge q)\wedge\sim(\sim p\wedge r)$

$= (p\wedge q)\wedge(p\vee\sim r)$

$= (p\wedge q)$

4. Let 3,6,9 ....upto 78th term and 5,9,13,17.....upto 59th term are two A.P., then the sum of common terms of two series is

Ans. **(2223)**

Sol. 3,6,9 ..... upto78th term

$t_{78} = 3+ (78 - 1)\ 3 = 234$

5,9,13,17.....upto 59th term

$T_{59} = 5 + (59-1)\ 4 = 237$

Now series of common term is

9,21,33.....

$t_n = 9 + (n-1)\ 12 \le 234$

$12n \le 237$

$n \le 19.75 \Rightarrow n = 19$

$\therefore S_{19} = \frac{19}{2}[18+(19-1)12] = 19\times117 = 2223$

5. The sum of series $1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}$ .....upto infinite terms is

(1) $\frac{287}{125}$ (2) $\frac{286}{125}$ (3) $\frac{288}{125}$ (4) $\frac{289}{125}$

Ans. **(3)**

Sol. Let $S = 1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\ldots\ldots\ldots\ldots$ (1)

$\frac{S}{6} = \frac{1}{6}+\frac{5}{6^2}+\frac{12}{6^3}+\ldots\ldots\ldots\ldots$ (2)

Equation (1) – (2)

$\frac{5S}{6} = 1+\frac{4}{6}+\frac{7}{6^2}+\frac{10}{6^3}+\ldots\ldots\ldots\ldots$ (3)

$\frac{5S}{36} = \frac{1}{6}+\frac{4}{6^2}+\frac{7}{6^3}+\ldots\ldots\ldots\ldots$ (4)

Equation (3) – (4)

$\frac{25S}{36} = 1+\frac{3}{6}+\frac{3}{6^2}+\frac{3}{6^3}+\ldots\ldots\ldots\ldots$

$= 1+\frac{\frac{3}{6}}{1-\frac{1}{6}} = 1+\frac{3}{5}$

$\frac{25S}{36} = \frac{8}{5} \Rightarrow S = \frac{288}{125}$

6. The number of solution of equation $\sin x = \cos^2 x$ in the interval (0,10) is

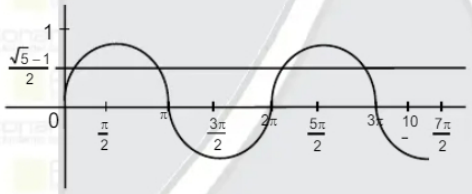
Ans. **(04.00)**

Sol. Given equation $\sin x = \cos^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1\pm\sqrt{5}}{2}$

$\Rightarrow \sin x = \frac{\sqrt{5}-1}{2}$



$\therefore$ Number of solution = 4

7. Let α be the root of equation $x^4+x^2+1=0$, then the value of $\alpha^{1011}+\alpha^{2022}-\alpha^{3033}$ is :

(1) $1+2\alpha$ (2) α (3) 1 (4) $1+\alpha$

Ans. **(3)**

Sol. $x^4+x^2+1=0$

α is a root $\therefore \alpha^4+\alpha^2+1=0$

$\Rightarrow \alpha^2 = \omega$ or ω^2

Now $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$

$= \alpha.(\alpha^2)^{505} + (\alpha^2)^{1011} - \alpha.(\alpha^2)^{1516}$

$= \alpha.\omega^{505} + \omega^{1011} - \alpha.\omega^{1516}$

$= \alpha\omega + 1 - \alpha\omega$

$= 1$

8. If $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = I - {}^5C_1 (\text{adj } A) + {}^5C_2(\text{adj}A)^2 - {}^5C_3(\text{adj}A)^3 + {}^5C_4(\text{adj}A)^4 - {}^5C_5(\text{adj}A)^5$, then the sum of elements of B is

(1) –5 (2) –7 (3) –6 (4) –4

Ans. **(2)**

Sol. $B = (I - (\text{adj } A)^5 = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\right)^5 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}^5 = C^5$

$C^2 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$C^4 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$B = C^4C = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$

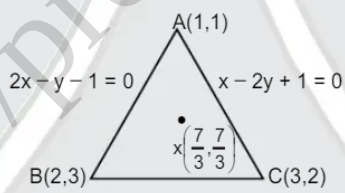
Sum of elements = (–1) + (–5) + (–1) = – 7

9 The two sides of a triangle are $2x - y - 1 = 0$ and $x - 2y + 1 = 0$. If orthocentre of triangle is $\left(\frac{7}{3}, \frac{7}{3}\right)$, then distance of origin from centroid of triangle is

(1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) $3\sqrt{2}$ (4) $4\sqrt{2}$

Ans. **(2)**

Sol.



altitude through B is $2x + y + \lambda = 0$

$\Rightarrow \frac{14}{3} + \frac{7}{3} + \lambda = 0$

$\lambda = -7$

$\therefore$ altitude is $2x + y - 7 = 0$

equation of AB is $2x - y - 1 = 0$

$\therefore$ B (2, 3), similarly C(3,2)

$\therefore$ centroid G (2,2)

$\therefore$ OG = $\sqrt{4+4} = 2\sqrt{2}$