Hence $|\alpha^8 + \beta^8| = 2|\alpha^2|^4 = 2\left(\sqrt{1+4}\right)^4 = 50$

2. Domain of function $f(x) = \cos^{-1}\left[\dfrac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right]$ is

(1) $\left(-\infty,-\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}},\infty\right)$ (2) $\left(-\infty,-\frac{1}{2}\right] \cup \{0\} \cup \left[\frac{1}{2},\infty\right)$

(3) $\left(-\infty,-\frac{1}{\sqrt{2}}\right] \cup \{0\} \cup \left[\frac{1}{\sqrt{2}},\infty\right)$ (4) $\left(-\infty,\frac{1}{\sqrt{2}}\right]$

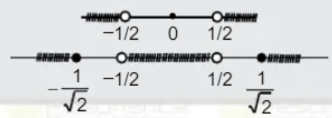
Ans. **(3)**

Sol. $-1 \le \dfrac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \le 1$

$\Rightarrow -\frac{\pi}{2} \le \sin^{-1}\left(\frac{1}{4x^2-1}\right) \le \frac{\pi}{2}$

$\Rightarrow -1 \le \frac{1}{4x^2-1} \le 1$

$\Rightarrow 0 \le \frac{x^2}{(2x-1)(2x+1)}$ and $\frac{(\sqrt{2}x-1)(\sqrt{2}x-1)}{(2x-1)(2x+1)} \ge 0$



Domain $\left(-\infty,-\frac{1}{\sqrt{2}}\right] \cup \{0\} \cup \left[\frac{1}{\sqrt{2}},\infty\right)$

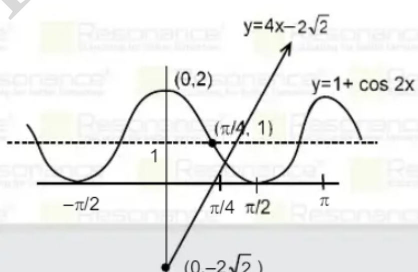
3. Total number of solution of the equation $2\theta - \cos^2\theta - \sqrt{2} = 0$ is/are

(1) 1 (2) 2 (3) 3 (4) 0

Ans. **(1)**

Sol. $4\theta - (1+\cos 2\theta) - 2\sqrt{2} = 0$

$1 + \cos 2\theta = 4\theta - 2\sqrt{2}$



by graph clearly only one solution

4. If $x\frac{dy}{dx} - y = \sqrt{y^2+16x^2}$ and $y(1) = 3$, then the value of $y(2)$ is

Ans. **(15.00)**

Sol. $\frac{dy}{dx} = \frac{y+\sqrt{y^2+16x^2}}{x}$

Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx+\sqrt{v^2x^2+16x^2}}{x} = v + \sqrt{v^2+16}$

$$\int \frac{dv}{\sqrt{v^2+16}} = \int \frac{dx}{x}$$

$$\ell n\left|v+\sqrt{v^2+16}\right| = \ell nx + \ell nc$$

$$\frac{y}{x}+\frac{\sqrt{y^2+16x^2}}{x} = cx$$

$$y + \sqrt{y^2+16x^2} = cx^2$$

y(1) = 3

$\Rightarrow$ c = 8

at x = 2 $\quad y+\sqrt{y^2+64} = 32$

$y^2 + 64 = (32 - y)^2$

$64(1+y) = 32 \times 32$

y = 15

5. The mirror image of point (2,3,7) in the plane 3x – y + 4z = 2 is (a,b,c) then the value of (2a + b + 2c) is

(1) 6 (2) – 6 (3) –8 (4) 12

Ans. (3)

Sol. The equation of plane is 3x – y + 4z = 2

the mirror image of point (2,3,7) in given plane is

$$\frac{x-2}{3} = \frac{y-3}{-1} = \frac{z-7}{4} = -\frac{2(6-3+28-2)}{(9+1+16)}$$

$$\frac{x-2}{3} = \frac{y-3}{-1} = \frac{z-7}{4} = -\frac{29}{13}$$

$$\frac{x-2}{3} = -\frac{29}{13} \Rightarrow x = \frac{-29\times 3}{13}+2 = \frac{-61}{13}$$

$$\frac{y-3}{-1} = \frac{-29}{13} \Rightarrow y = \frac{29}{13}+3 = \frac{68}{13}$$

$$\frac{z-7}{4} = \frac{-29}{13} \Rightarrow z = \frac{-29\times 4}{13}+7 = -\frac{25}{13}$$

the mirror image of point (2,3,7) in given plane is $\left(-\frac{61}{13},\frac{68}{13},-\frac{25}{13}\right)$

the value of (2a + b + 2c) is –6

6. The constant term in the expansion of $\left(3x^3+2x^2-\frac{5}{x^5}\right)^{10}$ is

(1) $\frac{10!}{6!3!}(3)^2(2)^5(-5)^3$ (2) $\frac{10!}{7!3!}(3)^2(-5)^3$ (3) $\frac{10!}{6!3!}(3)(2)^6(-5)^3$ (4) $\frac{10!}{2!2!6!}(3)^2(2)^6(5)^2$

Ans. (3)

Sol. $\left(3x^3+2x^2-\frac{5}{x^5}\right)^{10} = x^{-50}(3x^8+2x^7-5)^{10}$

constant term in $x^{-50}(3x^8+2x^7-5)^{10}$

coefficient of x^{50} in $(3x^8+2x^7-5)^{10}$

coefficient of x^{50} in $\sum \frac{10!}{r_1!r_2!r_3!}(3)^{r_1}(2)^{r_2}(-5)^{r_3} \cdot x^{8r_1+7r_2}$

such that $r_1 + r_2 + r_3 = 10$ and $8r_1 + 7r_2 = 50$

$\Rightarrow r_1 = 1; r_2 = 6$ and $r_3 = 3$

Hence constant term is = $\frac{10!}{1!6!3!}(3)(2)^6(-5)^3$

7. The value of $\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$ is (where [·] denotes G.I.F.)

(1) 1 (2) 2 (3) 0 (4) 3

Ans. (3)

Sol. $\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$

$$= \int_0^2 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx + \int_2^4 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx + \int_4^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$$

$$= \int_0^2 \cos(\pi x - 0)dx + \int_2^4 \cos(\pi x - \pi)dx + \int_4^5 \cos(\pi x - 2\pi)dx$$

$\int_0 \cos(\pi x - 0)dx + \int_2 \cos(\pi x - \pi)dx + \int_4 \cos(\pi x - 2\pi)dx$

$= \left[\frac{\sin \pi x}{\pi}\right]_0^2 - \left[\frac{\sin \pi x}{\pi}\right]_2^4 + \left[\frac{\sin \pi x}{\pi}\right]_4^5$

$= 0$

8. The mean and variance of x_1, x_2, x_3, x_4, x_5, is $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If mean and variance of first four observation is $\frac{7}{2}$ and a, then the value of $4a + x_5$ is

Ans. **(15.00)**

Sol. $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$

$\Rightarrow$ $x_1 + x_2 + x_3 + x_4 + x_5 = 24$ ...(1)

and $\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$

$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} = \frac{194}{25} + \frac{576}{25} = \frac{770}{25}$

$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 154$ ....(2)

Now $\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$

$\Rightarrow$ $x_1 + x_2 + x_3 + x_4 = 14$ ....(3)

$\Rightarrow$ $\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \left(\frac{7}{2}\right)^2 = a$

$\frac{154 - x_5^2}{4} - \frac{49}{4} = a$

$\frac{154}{4} - \frac{49}{4} - \frac{x_5^2}{4} = a$

$4a + x_5^2 = 105$

from equation (1) and (3)

$x_5 = 10$, $4a = 5$

$\Rightarrow$ $4a + x_5 = 5 + 10 = 15$

9. If $A = [a_{ij}]$ is a 3×3 order matrix, such that $a_{ij} = 2^{j-i}$ then $A^2 + A^3 + ... + A^{10}$ is equal to

(1) $\frac{3(3^9 - 1)}{2}A$ (2) $\frac{3(3^9 + 1)}{2}A$ (3) $\frac{3(3^{10} - 1)}{2}A$ (4) $\frac{3(3^{10} + 1)}{2}A$

Ans. **(1)**

Sol. $A = \begin{bmatrix} 1 & 2 & 2^2 \\ 2^{-1} & 1 & 2 \\ 2^{-2} & 2^{-1} & 1 \end{bmatrix} \Rightarrow A^2 = 3A, A^3 = 3^2A, \ldots\ldots\ldots\ldots\ldots, A^{10} = 3^9A$

Now,

$A^2 + A^3 \ldots\ldots + A^{10}$

$= 3A + 3^2A \ldots\ldots\ldots + 3^9A$

$= (3 + 3^2 + 3^3 \ldots\ldots\ldots + 3^9)A$

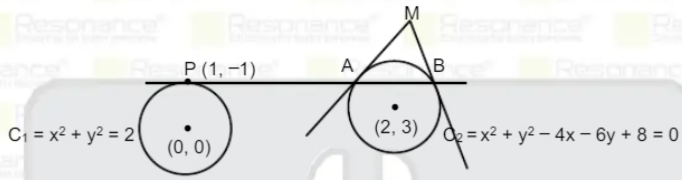
$= \frac{3(3^9 - 1)}{2}A$

10. If tangent drawn to a circle $x^2 + y^2 = 2$ at point $(1, -1)$ intersect another circle $(x - 2)^2 + (y - 3)^2 = 5$ at two point A and B. If tangents drawn to this circle intersect at point M, then the area of ΔAMB is

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{6}$ (4) $\frac{1}{12}$

Ans. (3)

Sol.



Equation of tangent to circle $x^2 + y^2 = 2$ at point $P(1, -1)$ is

$T = 0$

$x(1) + y(-1) = 2$

$x - y = 2$ ...(1)

Let point M(h, k)

Equation of chord of contact of circle (2) drawn from point M(h, k) is

$T = 0$

$hx + ky - 2(x + h) - 3(y + k) + 8 = 0$

$(h - 2)x + (k - 3)y - 2h - 3k + 8 = 0$ ...(2)

by comparing (1) and (2)

$\frac{h-2}{1} = \frac{k-3}{-1} = \frac{2h+3k-8}{2}$

$-h + 2 = k - 3$ and $2h - 4 = 2h + 3k - 8$

$h + k = 5$ $3k = 4$

$k = \frac{4}{3}$

$h = 5 - \frac{4}{3} = \frac{11}{3}$

So, $M\left(\frac{11}{3}, \frac{4}{3}\right)$

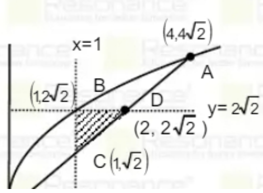
11. The area bounded between curves $y^2 = 8x$ and $y = \sqrt{2}x$, excluding the area of triangle formed by $y = \sqrt{2}x$, $x = 1$ and $y = 2\sqrt{2}$ is

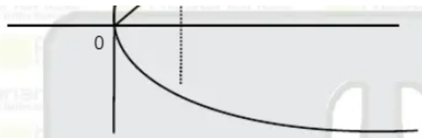
(1) $\left(\frac{8\sqrt{2}}{3} - \frac{1}{\sqrt{2}}\right)$ (2) $\left(\frac{8\sqrt{2}}{3} - \frac{1}{2}\right)$

(3) $\left(\frac{8\sqrt{2}}{3} - 1\right)$ (4) $\left(8\sqrt{3} - 4\right)$

Ans. (1)

Sol.





area of $\Delta BCD = \frac{1}{2}(\sqrt{2})(1) = \frac{1}{\sqrt{2}}$

area between curves = $\int_0^4 (2\sqrt{2}x^{\frac{1}{2}} - \sqrt{2}x)dx$

$= \left(\frac{2\sqrt{2}x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{\sqrt{2}}\right)\Bigg|_0^4 \qquad = \left(\frac{4}{3}\sqrt{2}x^{\frac{3}{2}} - \frac{1}{\sqrt{2}}x^2\right)\Bigg|_0^4$

$= \frac{32\sqrt{2}}{3} - 8\sqrt{2} = \frac{8\sqrt{2}}{3}$

Required area = $\left(\frac{8\sqrt{2}}{3} - \frac{1}{\sqrt{2}}\right)$

Length of tangent drawn from M to circle (2)

$L = \sqrt{S_1} = \sqrt{\left(\frac{11}{3}\right)^2 + \left(\frac{4}{3}\right)^2 - \frac{44}{3} - \frac{24}{3} + 8}$

$= \sqrt{\frac{121}{9} + \frac{16}{9} - \frac{68}{3} + 8}$

$= \sqrt{\frac{137}{9} - \frac{44}{3}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

radius of circle (2) is $R = \sqrt{5}$

Now area of $\Delta AMB = \frac{RL^3}{R^2 + L^2} = \frac{\sqrt{5} \times \frac{5\sqrt{5}}{27}}{5 + \frac{5}{9}}$

$= \frac{25}{27} \times \frac{9}{50} = \frac{1}{6}$ sq. unit

12. Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, be a hyperbola with eccentricity $\frac{\sqrt{11}}{2}$. If sum of the length of transverse axis and conjugate axis is $2\sqrt{2} + \sqrt{14}$ then value of $a^2 + b^2$ is

(1) $\frac{11}{4}$ (2) $\frac{11}{2}$ (3) $\frac{9}{2}$ (4) $\frac{11}{3}$

Ans. (2)

Sol. $e^2 = 1 + \frac{b^2}{a^2} \Rightarrow \frac{11}{4} = 1 + \frac{b^2}{a^2}$

$\Rightarrow 7a^2 = 4b^2$

$\Rightarrow b^2 = \frac{7}{4}a^2$ ....(1)

So hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{\left(\frac{7}{4}a^2\right)} = 1$

Sum of lengths of transverse axis and conjugate axis

$= 2a + \sqrt{7}a = 2\sqrt{2} + \sqrt{14}$

$(2 + \sqrt{7})a = \sqrt{2}(2 + \sqrt{7})$

$\Rightarrow a = \sqrt{2}$

and $b^2 = \frac{7}{2}$

Hence $a^2 + b^2 = 2 + \frac{7}{2} = \frac{11}{2}$

13. If $(p \wedge q) \Delta [(p \vee q) \Rightarrow q]$ is an tautology and $\Delta \in \{\Rightarrow, \Leftrightarrow, \wedge, \vee\}$ then Δ is

(1) $\Rightarrow$ (2) $\Leftrightarrow$ (3) $\wedge$ (4) $\vee$

Ans. **(1)**

Sol.

p	q	$p \wedge q$	$(p \vee q) \Rightarrow q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

14. If A is a square matrix using from first 10 prime numbers, then the probability that A is a non invertible matrix, is

Ans. **(0.019)**

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

so total number of matrix formed = 10^4

For non-invertible matrix $|A| = ad - bc = 0 \Rightarrow ad = bc$

Case I $a \neq d$ Case II $a = d$

$(10 \times 9) \times 2!$ 10×1

So required probability = $\frac{10 \times 9 \times 2! + 10}{10^4} = \frac{19}{1000} = 0.019$

15. Find the value of $\lim_{n \to \infty} 6 \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right)\right\}$ is

(1) 1 (2) 2 (3) 3 (4) 6

Ans. **(3)**

Sol. $\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right) = \sum_{r=1}^{n} \tan^{-1}\left(\frac{(r+2) - (r+1)}{1 + (r+1)(r+2)}\right)$

$\sum_{r=1}^{n} \tan^{-1}(r+2) - \tan^{-1}(r+1)$

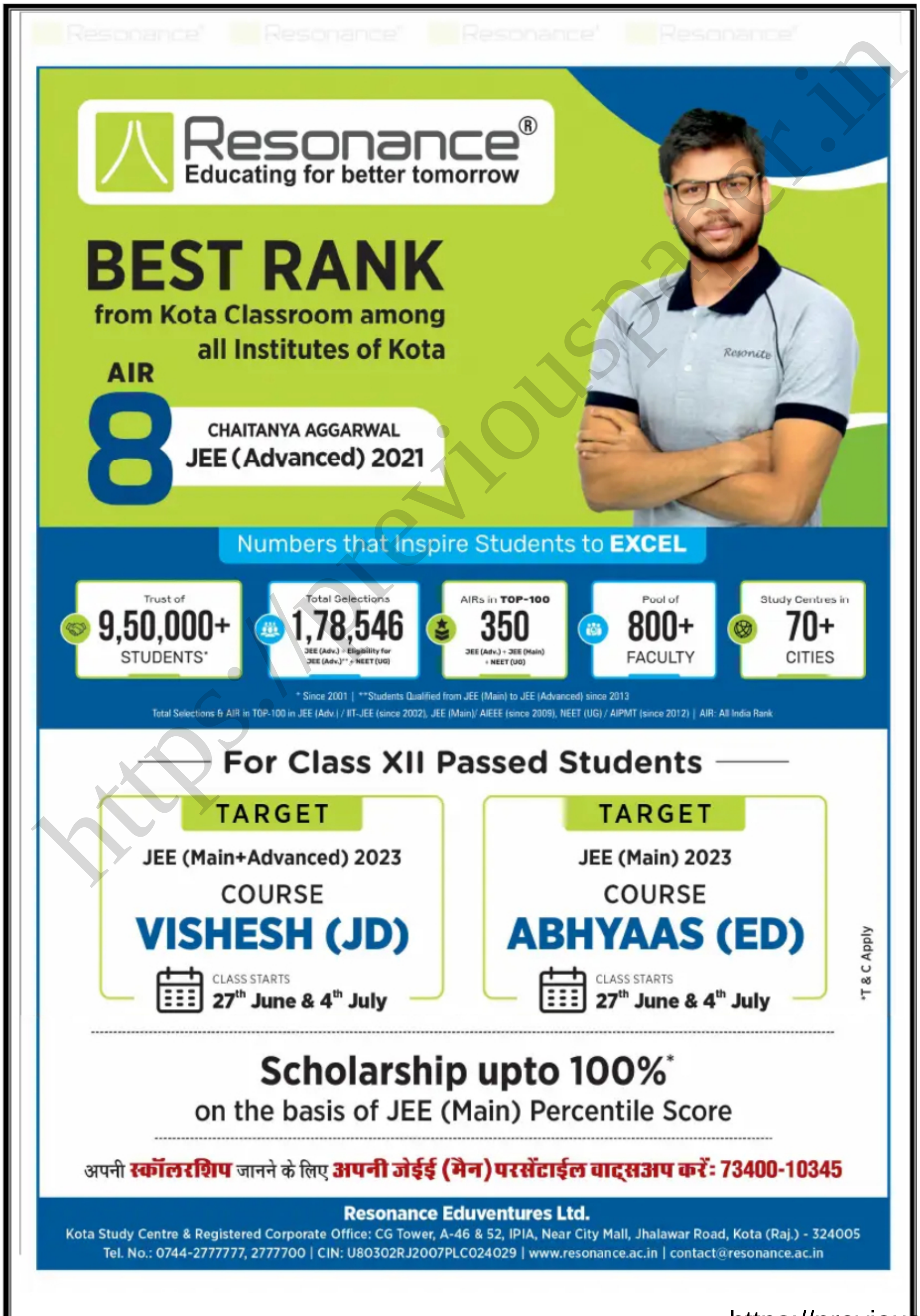
$= (\tan^{-1}3 - \tan^{-1}2) + (\tan^{-1}4 - \tan^{-1}3) + (\tan^{-1}5 - \tan^{-1}4) + \ldots\ldots + (\tan^{-1}(n+2) - \tan^{-1}(n+1))$

$= \tan^{-1}(n+2) - \tan^{-1}(2)$

$= \tan^{-1}\left(\frac{n}{1 + 2(n+2)}\right)$

$\Rightarrow 6 \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right)\right\} = \frac{6n}{5 + 2n}$

$\lim_{n \to \infty} 6 \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right)\right\} = \frac{6}{2} = 3$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