Sol. $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)\right)$

$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)\right)$

$\cot\left(\sum_{n=1}^{50}\left[\tan^{-1}(n+1)-\tan^{-1}n\right]\right)$

$\cot\left[\left(\tan^{-1}(2)-\tan^{-1}(1)\right)+\left(\tan^{-1}(3)-\tan^{-1}(2)\right)+.....+\left(\tan^{-1}(51)-\tan^{-1}(50)\right)\right]$

$\cot\left[\tan^{-1}51-\tan^{-1}1\right]$

$\cot\left[\tan^{-1}\left(\frac{50}{52}\right)\right]=\frac{26}{25}$

2. A and B are two square matrix of order 3. Where AB = I and $|A|=\frac{1}{8}$ then the value of |adj (B(adj2A))| is

Ans. **64.00**

Sol. AB = I

$|A|\,|B| = 1$

So $|B| = 8$

$|\text{adj}(B(\text{adj}2A))| = |B(\text{adj}2A)|^2$

$= |B|^2\,|\text{adj }2A|^2 = 64\times|2A|^4$

$= 64\times 8^4\times|A|^4 = 64$

3. If $a_1, a_2, \ldots\ldots\ldots, a_{10}$ and $b_1, b_2, \ldots\ldots\ldots b_{10}$ are in A.P. and $a_1 = 2$, $a_{10} = 3$, also $a_1b_1 = 1 = a_{10}b_{10}$, then the value of $a_4 b_4$ is

(1) $\frac{14}{27}$ (2) $\frac{28}{27}$ (3) $\frac{7}{27}$ (4) $\frac{15}{27}$

Ans. **(2)**

Sol. $\frac{a_{10}-a_1}{9}=\frac{3-2}{9}=d$

$d=\frac{1}{9}$

$a_4 = 2 + 3d = 2+\frac{1}{3} = 7/3$

Also $b_1=\frac{1}{2}$ and $b_{10}=\frac{1}{3}$

Now $d_1=\frac{b_{10}-b_1}{9}=\frac{\frac{1}{3}-\frac{1}{2}}{9}=\frac{-1}{54}$

$b_4=\frac{1}{2}-\frac{3}{54}=\frac{9-1}{18}=\frac{8}{18}=\frac{4}{9}$

Now then the value of $a_4 b_4 = \frac{7}{3}\times\frac{4}{9}=\frac{28}{27}$

4. sin 36° is root of equation whose coefficients are rational is given by

(1) $16x^4 + 20x^2 - 5 = 0$ (2) $16x^4 + 20x^2 + 5 = 0$

(3) $16x^4 - 20x^2 + 5 = 0$ (4) $8x^4 - 20x^2 + 5 = 0$

Ans **(3)**

Sol. $\sin 36° = x = \frac{\sqrt{10-2\sqrt{5}}}{4}$

$16x^2 = 10-2\sqrt{5}$

$(16x^2 - 10)^2 = 20$

$(8x^2 - 5)^2 = 5$

$64x^4 + 25 - 80x^2 - 5 = 0$

$64x^4 - 80x^2 + 20 = 0$

$16x^4 - 20x^2 + 5 = 0$

5. If $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + ... + \infty$, then the value of 4S is

(1) $\left(\frac{7}{3}\right)^2$ (2) $\left(\frac{7}{3}\right)^3$ (3) $\left(\frac{7}{3}\right)^4$ (4) $\left(\frac{7}{3}\right)^5$

Ans. (2)

Sol. $S = \frac{2}{1} + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + ...\infty$

$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + ...\infty$

$\frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + ...\infty$

$\frac{6S}{7} \times \frac{1}{7} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} .....\infty$

$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} .....\infty = \frac{2}{1 - \frac{1}{7}} = \frac{14}{6} = \frac{7}{3}$

$S = \frac{7}{3} \times \frac{49}{36}$

then the value of $4S = \frac{343}{3 \times 36} \times 4 = \left(\frac{7}{3}\right)^3$

6. If the mean and variance of observations 4, 5, 6, 6, 7, 8, x, y where x<y are 6 and $\frac{9}{4}$ respectively, then the value of $x^4 + y^2$ is

Ans. 320.00

Sol. $\bar{x} = \frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow x + y = 12$..........(1)

$\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \frac{9}{4} = \frac{16+25+36+36+49+64+x^2+y^2}{8} - 36$

$\Rightarrow x^2 + y^2 = 80$ ....(2)

from (1) and (2) x= 4, y = 8

$x^4 + y^2 = 256 + 64 = 320$

7. Which of the following is tautology?

(1) $(\sim p \wedge q) \vee (p \vee \sim p)$ (2) $(p \wedge q) \wedge (p \vee \sim p)$

(3) $(\sim p \wedge q) \wedge (p \vee \sim p)$ (4) $(\sim p \wedge \sim q) \vee (p \wedge \sim p)$

Ans. (1)

Sol. $p \vee \sim p$ is tautology with this any 'or' gives tautology so (1) is Answer

8. $y(x) = (x^x)^x$ then the value of $\frac{d^2y}{dx^2} + 20$ at x = 1 is

Ans. 24.00

Sol. $y(x) = (x^x)^x$ at x = 1 ⇒ y = 1

$\ell ny = x^2 \ell nx$

$\frac{1}{y} y' = x^2 \frac{1}{x} + 2x.\ln x$

$y' = y (x + 2x \ln x)$ ....(1)

$(y')_{x=1} =$ 1(1+0) = 1

$y'' = y\left(1 + \frac{2x}{x} + 2\ln x\right) + (x + 2x\ln x)y'$

$(y'')_{x=1} = 1 (1 + 2 + 0) + (1 \times 1) = 4$

So, $\frac{d^2y}{dx^2} + 20 = 24$

9. If a curve $y = f(x)$ satisfies the differential equation $(\tan^{-1} y - x)dy = (1 + y^2)dx$ passes through point (1,0), then the value of x at $y = \tan 1$ is

(1) 2 (2) 3 (3) 3 (4) 4

Ans. (1)

Sol. $(\tan^{-1} y - x)dy = (1 + y^2)dx$

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$

I.F. $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$x.e^{\tan^{-1}y} = \int e^{\tan^{-1}y} \frac{\tan^{-1}y}{1+y^2} dy$

Let $\tan^{-1}y = t$

$\frac{1}{1+y^2}dy = dt$

$x.e^{\tan^{-1}y} = \int e^t.t\,dt$

$xe^{\tan^{-1}y} = t.e^t + \int 1.e^t\,dt$

$x.e^{\tan^{-1}y} = t.e^t + e^t + C$

$x.e^{\tan^{-1}y} = e^t(t+1)+C$

$x.e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y+1)+C$ passes (1, 0)

$1 = 1(1) + C$

$x.e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y+1)$

$x.e. = e(1 + 1)$

$x = 2$

10. The shortest distance between lines $\frac{x-3}{3} = \frac{y-2}{2} = \frac{z-1}{-1}$ and $\frac{x+3}{3} = \frac{y-6}{1} = \frac{z-5}{2}$ is

(1) $\frac{78}{\sqrt{115}}$ (2) $\frac{79}{\sqrt{115}}$ (3) $\frac{20}{\sqrt{115}}$ (4) $\frac{1}{\sqrt{115}}$

Ans. (1)

Sol. Normal vector of both lines

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 3 & 1 & 2 \end{vmatrix} = \hat{i}(5) - \hat{j}(9) + \hat{k}(-3)$

$= 5\hat{i} - 9\hat{j} - 3\hat{k}$

Now vector joining (3, 2, 1) and (–3,6, 5) is

$(-6\hat{i} + 4\hat{j} + 4\hat{k})$

Now, shortest distance

$= \left| \frac{(5\hat{i} - 9\hat{j} - 3\hat{k}).(-6\hat{i} + 4\hat{j} + 4\hat{k})}{|5\hat{i} - 9\hat{j} - 3\hat{k}|} \right|$

$= \left| \frac{-30 - 36 - 12}{\sqrt{25+81+9}} \right|$

$= \frac{78}{\sqrt{115}}$

11. A matrix $[a_{ij}]_{2\times2}$ where $a_{ij} \in \{0, 1, 2, 3, 4, 5\}$ is formed such that sum of its elements is prime number p and $p \in (2, 8)$, then number of possible matrices

(1) 187 (2) 180 (3) 185 (4) 182

Ans **(2)**

Sol. Let matrix is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then a + b + c + d = 3 or 5 or 7

for 3 $\Rightarrow {}^{4+3-1}C_3 = 20$

For 5 $\Rightarrow {}^{5+4-1}C_3 = 56$

for 7 $\Rightarrow$ coeff x^7 $(1 + x + x^2 + \ldots + x^5)^4$

$= (1-x^6)^4 (1-x)^{-4}$

$= ({}^4C_0 - {}^4C_1 x^6)(1-x)^{-4}$

$= {}^{4+7-1}C_7 - {}^4C_1 \times {}^{4+1-1}C_1$

$= \dfrac{10.9.8}{3.2} - 4 \times 4 = 120 - 16$

So total matrix = 20 + 56 + 104 = 180

12. If vertex of parabola $x^2 + ay^2 + bxy + cx + dy + e = 0$ is at (5,4) and its directrix is $3x + y - 29 = 0$ then the value of $a + b + c + d + e$ is

(1) 576 (2) –576 (3) –575 (4) 575

Ans (2)

Sol. Tangent at vertex is $3x + y + p = 0$, It passes through (5, 4), then $p = -19$

So tangent at vertex $3x + y - 19 = 0$

Equation of Axis parabola $\Rightarrow x - 3y + q = 0$, It passes through (5,4),

$5 - 12 + q = 0$

$q = 7$

distance between vertex and directrix = a = $\left|\dfrac{15 + 4 - 29}{\sqrt{10}}\right|$

$4a = 4\sqrt{10}$

Now equation of parabola is

$\left(\dfrac{x - 3y + 7}{\sqrt{10}}\right)^2 = -4\sqrt{10}\left(\dfrac{3x + y - 19}{\sqrt{10}}\right)$

$(x - 3y + 7)^2 + 40(3x + y - 19) = 0$

Sum of coefficient for x = y = 1

$= 25 + 40(-15)$

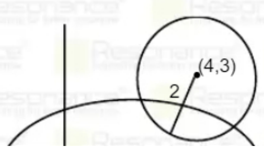
So $a + b + c + d + e = -575 - 1 = -576$

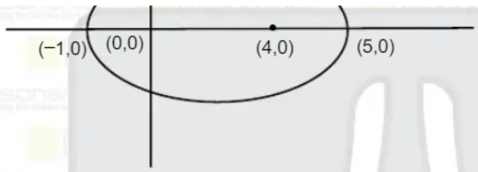
13. The number of complex numbers z satisfying $|z - (4 + 3i)| = 2$ and $|z| + |z - 4| = 6$ is

(1) 2 (2) 3 (3) 4 (4) 5

Ans. **(1)**

Sol.

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(0,0)
(4,0)
(5,0)