Sol. We known that, eccentricity of ellipse

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \frac{1}{4} = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow \frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$b^2 = \frac{15a^2}{16}$$

the point $P\left(-4\sqrt{\frac{2}{5}}, 3\right)$ lie on ellipse

so put $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{16 \times \frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{b^2} = 1$$

$$\because b^2 = \frac{15a^2}{16}$$

$$\Rightarrow \frac{32}{5a^2} + \frac{16 \times 9}{15a^2} = 1$$

$$\frac{16}{15}\left[\frac{3 \times 2 + 9}{a^2}\right] = 1$$

$$\frac{16}{15}\left[\frac{15}{a^2}\right] = 1$$

$a^2 = 16$

$b^2 = 15$

then the value of $a^2 + b^2 = 31$

2. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is

(1) –1 (2) $-\frac{1}{2}$ (3) $-\frac{1}{3}$ (4) $-\frac{1}{4}$

Ans. **(2)**

Sol. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$\Rightarrow \frac{\cos\left(\frac{2\pi}{7} + \frac{2}{2} \times \frac{2\pi}{7}\right).\sin\left(\frac{3}{2}.\frac{2\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}$$

$$\Rightarrow \frac{-\cos\frac{3\pi}{7}.\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}$$

$$\Rightarrow \frac{-2\sin\frac{3\pi}{7}.\cos\frac{3\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$\Rightarrow -\frac{1}{2}\frac{\sin\frac{6\pi}{7}}{\sin\frac{\pi}{7}}$$

$\Rightarrow -\frac{1}{2}\frac{\sin\left(\pi-\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} = -\frac{1}{2}$

3. The value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to

(1) $\frac{11\pi}{12}$ (2) $\frac{\pi}{4}$ (3) $\frac{5\pi}{3}$ (4) $\frac{7\pi}{6}$

Ans. **(1)**

Sol. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) + \cos^{-1}\left(-\cos\frac{\pi}{6}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$= \frac{\pi}{3}+\pi-\frac{\pi}{6}-\frac{\pi}{4}$

$= \frac{11\pi}{12}$

4. Five numbers x_1, x_2, x_3, x_4, x_5 are selected randomly from set {1,2,3,- - -- , 18} such that $x_1 < x_2 < x_3 < x_4 < x_5$ then the probability that $x_2 = 7$ and $x_4 = 11$ is.

(1) $\frac{1}{34}$ (2) $\frac{1}{68}$ (3) $\frac{1}{36}$ (4) $\frac{1}{17}$

Ans. **(2)**

Sol. Total ways = ${}^{18}C_5$

Since $x_1 \in [1,6]$, $x_3 \in [8, 10]$, $x_5 \in [12, 18]$

$\Rightarrow$ Favourable ways = ${}^6C_1 . {}^3C_1 . {}^7C_1$

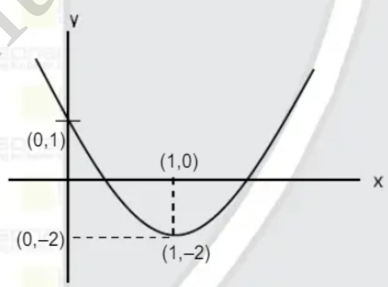
Probability = $\frac{{}^6C_1\,{}^3C_1.{}^7C_1}{{}^{18}C_5} = \frac{6\times3\times7}{\left(\frac{18\times17\times16\times15\times14}{5\times4\times3\times2\times1}\right)} = \frac{1}{68}$

5. Number of real roots of equation $x^4 - 4x + 1 = 0$ is

(1) 4 (2) 2 (3) 1 (4) 0

Ans. **(2)**

Sol. Let $f(x) = x^4 - 4x + 1$



$f'(x) = 4x^3 - 4$

$f'(x) = 0 \Rightarrow x = 1$

$f(1) = -2$ and $f(0) = 1$

Two real roots

6. There are 16 identical cubes in which 5 are red and 11 are blue, the number of ways in which these cubes can be arranged in a row such that between any two red cubes there are atleast two blue cubes, is

Ans. **(56)**

Sol. $x_1Rx_2Rx_3Rx_4Rx_5Rx_6$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

≥ 0 ≥ 2 ≥ 2 ≥ 2 ≥ 2 ≥ 0

where $x_1, x_6 \geq 0$ and $x_2, x_3, x_4, x_5 \geq 2$

let $x_2 = t_2 + 2$, $x_3 = t_3 + 2$, $x_4 = t_4 + 2$

let $x_2 = t_2 + 2$, $x_3 = t_3 + 2$, $x_4 = t_4 + 2$

$x_5 = t_5 + 2$

where $t_2, t_3, t_4, t_5 \geq 0$

equation $x_1 + t_2 + 2 + t_3 + 2 + t_4 + 2 + t_5 + 2 + x_6 = 11$

$\Rightarrow x_1 + t_2 + t_3 + t_4 + t_5 + x_6 = 3$

number of non-negative integral solutions

$= {}^{3+6-1}C_{6-1} = {}^8C_5 = {}^8C_3 = 56$

7. If $f(x) = \dfrac{2.e^{2x}}{e^{2x}+e}$ then the value of $f\left(\dfrac{1}{100}\right)+f\left(\dfrac{2}{100}\right)+....+f\left(\dfrac{99}{100}\right)$ is

(1) 98 (2) 99 (3) 100 (4) 97

Ans. **(2)**

Sol. $f(1-x) + f(x) = \dfrac{2.e^{2(1-x)}}{e^{2(1-x)}+e} + \dfrac{2.e^{2x}}{e^{2x}+e}$

$= \dfrac{2.e^2.e^{-2x}}{e^2.e^{-2x}+e} + \dfrac{2.e^{2x}}{e^{2x}+e} = \dfrac{2e}{e+e^{2x}} + \dfrac{2e^{2x}}{e^{2x}+e}$

$f(1-x) + f(x) = 2$

Now $\left(f\left(\dfrac{1}{100}\right)+f\left(\dfrac{99}{100}\right)\right)+\left(f\left(\dfrac{2}{100}\right)+f\left(\dfrac{98}{100}\right)\right)+.....+\left(f\left(\dfrac{49}{100}\right)+f\left(\dfrac{51}{100}\right)\right)+f\left(\dfrac{50}{100}\right)$

$\Rightarrow \underbrace{(2+2+......+2)}_{49\text{ times}}+f\left(\dfrac{1}{2}\right) = 2\times 49 + \dfrac{2.e}{e+e}$

$= 98 + 1 = 99.$

8. In a binomial distribution n = 7 and P(x = 3) = 5P (x = 4), then the value of sum of mean and variance is

(1) $\dfrac{14}{36}$ (2) $\dfrac{77}{36}$ (3) $\dfrac{31}{36}$ (4) $\dfrac{35}{36}$

Ans. **(2)**

Sol. Given P(x = 3) = 5P(x = 4)

${}^7C_3\, P^3q^4 = 5({}^7C_4\, P4q^3)$

$q = 5P$ .....(1)

$p + q = 1$ .....(2)

$6p - 1 = 0$

$\Rightarrow p = \dfrac{1}{6}$

$q = \dfrac{5}{6}$

mean = np = $7.\dfrac{1}{6} = \dfrac{7}{6}$

variance = npq = $7.\dfrac{1}{6}.\dfrac{5}{6} = \dfrac{35}{36}$

sum of mean and variance = $\dfrac{7}{6}+\dfrac{35}{36}$

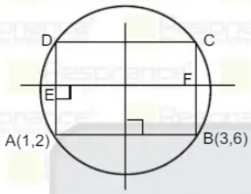
$= \dfrac{77}{36}$

9. A rectangle whose two adjacent vertices are (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of circle is $2x - y + 4 = 0$, then the area of the rectangle is

(1) 6 (2) 8 (3) 16 (4) 20

Ans. (3)

Sol.



Length of AB = $\sqrt{2^2+4^2} = 2\sqrt{5}$

Slope of AB = $\frac{6-2}{3-1} = 2$

$\Rightarrow$ AB || diameter $2x - y + 4 = 0$

AD = 2(AE) = $2.\frac{|2(1)-2+4|}{\sqrt{2^2+1^2}} = \frac{8}{\sqrt{5}}$

Area of rectangle = $\frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16$

10. If the coefficient of x^{10} in the expansion of $\left(\frac{x^{1/2}}{5^{1/4}}+\frac{5^{1/2}}{x^{1/2}}\right)^{60}$ is equal to $5^k.\ell$ where ℓ is co-prime with 5 then value of k is

Ans. (05.00)

Sol. $T_{r+1} = {}^{60}C_r = \left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r}\left(\frac{5^{1/2}}{x^{1/2}}\right)^{r}$

$\frac{60-r}{2} - \frac{r}{3} = 10$

$180 - 3r - 2r = 60$

$5r = 120, r = 24$

$T_{24+1} = {}^{60}C_{24}\left(\frac{1}{5^{1/4}}\right)^{36}\left(5^{1/2}\right)^{24} = {}^{60}C_{24} . 5^3$

$= {}^{60}C_{24} \times 5^3 = \frac{60!}{24!36!}\times 5^3$

Exponent of 5 in 60 ! = $\left[\frac{60}{5}\right]+\left[\frac{60}{25}\right]$ = 12 + 2 = 14

Exponent of 5 in 24! = $\left[\frac{24}{5}\right]$ = 4

Exponent of 5 in 36! = $\left[\frac{36}{5}\right]+\left[\frac{36}{25}\right]$ = 7 + 1 = 8

Exponent of 5 in ${}^{60}C_{24}$ = 14 – 4 – 8 = 2

k = 2 + 3 = 5

11. If the non-real roots of $z^2 = i\bar{z}$ the vertices of a polygon then the area of the polygon is

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{\sqrt{5}}{4}$

Ans. (2)

Sol. $z^2 = i\bar{z}$

$|z|^2 = |i\bar{z}|$

$|z| = 1$

$z^3 = i\bar{z}z$

$z^3 = e^{i\left(\frac{\pi}{2}+2n\pi\right)}$

$z = e^{i(4x+1)\frac{\pi}{6}}$ (where n = 0,1,2)

$z_1 = e^{i\frac{\pi}{6}}, e^{i\frac{5\pi}{6}}, -i$

$$\frac{1}{2}\begin{vmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & -1 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2\frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & -1 & 1 \end{vmatrix}$$

$\Rightarrow \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$

12. If $\frac{dy}{dx} + \frac{2^{x-y}(2^y-1)}{2^x-1} = 0$ and y(1) = 1 then the value of y(2) is

(1) $\log_4\left(\frac{3}{4}\right)$ (2) $\log_2\left(\frac{3}{4}\right)$ (3) $\log_2\left(\frac{4}{3}\right)$ (4) $\log_4\left(\frac{4}{3}\right)$

Ans. (3)

Sol. $\frac{2^y}{2^y-1}dy + \frac{2^x}{2^x-1}dx = 0$

$\ln(2^y - 1) + \ln(2^x - 1) = \ln c$

$(2^y - 1)(2^x - 1) = c$

given y(1) = 1

x = 1, y = 1

then c = 1

now x = 2

$(2^y - 1)(3) = 1$

$2^y - 1 = \frac{1}{3}$

$2^y = \frac{4}{3}$

$y = \log_2\left(\frac{4}{3}\right)$

13. If $\int \frac{e^x(x^2+1)}{(x+1)^2}dx = e^x.\ f(x) + C$ then the value of $\frac{d^3(f(x))}{dx^3}$ at x = 1 is

(1) $\frac{1}{4}$ (2) $\frac{3}{4}$ (3) $\frac{5}{4}$ (4) 1

Ans. (2)

Sol. $\int e^x.\left(\frac{x^2-1+2}{(x+1)^2}\right)dx$

$\int e^x.\left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right]dx$

$= e^x\ \frac{x-1}{x+1} + C$

$f(x) = \frac{x-1}{x+1}$

$\Rightarrow f(x) = 1 - \frac{2}{x+1}$

$f'(x) = \frac{2}{(x+1)^2}$

$f''(x) = -\frac{4}{(x+1)^3}$

$f'''(x) = \frac{12}{(x+1)^4}$

$f'''(1) = \frac{12}{\ldots} = \frac{3}{\ldots}$

$(x+1)^4 \quad 4$

14. Evaluate the $\int_{-2}^{2}\frac{|x^3+x|}{e^{x|x|}+1}dx$

(1) 2 (2) 4 (3) 6 (4) 8

Ans. **(3)**

Sol. Let $F(x) = \frac{|x^3+x|}{e^{x|x|}+1}$

So $F(-x) = \frac{|x^3+x|}{e^{-x|x|}+1}$

So $\int_{-2}^{2}F(x)dx = \int_{0}^{2}\{F(x)+F(-x)\}dx = \int_{0}^{2}|x^3+x|dx$

$= \int_{0}^{2}(x^3+x)dx = \left.\frac{x^4}{4}+\frac{x^2}{2}\right|_0^2$

$= \left(\frac{16}{4}+\frac{4}{2}\right)-(0-0) = 6$

15. The sides of triangles are $10 + x^2$, $10 + x^2$, $20 - 2x^2$, if the area of the triangle is maximum at x = k then the value of $3k^2$ is

(1) 3 (2) 6 (3) 7 (4) 10

Ans. **(4)**

Sol. $\Delta = \frac{1}{2}(20-2x^2)\sqrt{(10+x^2)^2-\left(\frac{20-2x^2}{2}\right)^2}$

$= (10-x^2)\sqrt{(10+x^2)^2-(10-x^2)^2}$

$= (10-x^2) . \sqrt{40x^2} = \sqrt{40}(10-x^3)$

$\frac{d\Delta}{dx} = \sqrt{40}(10-3x^2)$

Maxima $x = \frac{\sqrt{10}}{3} = k$

$3k^2 = 3\left(\frac{10}{3}\right) = 10$

16. If the sum of series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2061}+$ ------ upto 10 terms $= \frac{m}{n}$, where m & n are co-prime, then the value of m + n is

Ans. **16.00**

Sol. $S = \sum_{r=1}^{10}\frac{r}{4r^4+1} = \sum_{r=1}^{10}\frac{r}{(2r^2+1)^2-4r^2}$

$\sum_{r=1}^{10}\frac{r}{(2r^2+2r+1)(2r^2-2r+1)}$

$= \sum_{r=1}^{10}\left[\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right]\frac{1}{4}$

$= \frac{1}{4}\left[\left(1-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{25}\right)+----\right]$

$= \frac{1}{4}\left[1-\frac{1}{2(100)+2(10)+1}\right]$

$= \frac{1}{4}\left[1-\frac{1}{221}\right] = \frac{220}{221}\times\frac{1}{4} = \frac{55}{221} = \frac{5}{11} = \frac{m}{n}$

so, the value of m + n = 5 + 11 = 16

17. In an isosceles triangle ABC, with AB = AC, vertex A(6, 1), base BC is 2x + y = 4 and point B lies on x + 3y = 7, if centroid of triangle is (α,β) find the value of $15(\alpha+\beta)$ is

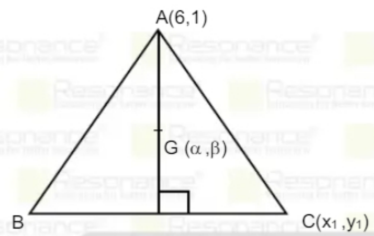
(1) 43 (2) 51 (3) 49 (4) 52

Ans. (2)

Sol. Let C be (x_1, y_1)

B is point of intersection of 2x + y = 4 and x + 3y = 7

$\Rightarrow$ B(1, 2)



$\because$ ABC is isosceles AG $\perp$ BC

(Slope of AG)(Slope of BC) = – 1

$\Rightarrow \left(\frac{\beta-1}{\alpha-6}\right)(-2) = -1 \Rightarrow \alpha - 2\beta = 4$ ....(1)

Now centroid = $(\alpha, \beta) \equiv \left(\frac{x_1+7}{3}, \frac{y_1+3}{3}\right)$

$\Rightarrow$ C $(x_1, y_1) \equiv (3\alpha - 7, 3\beta - 3)$ lies on 2x + y = 4

$\Rightarrow 2(3\alpha - 7) + 3\beta - 3 = 4$

$\Rightarrow 2\alpha + \beta = 7$ ....(2)

from (1) and (2)

$\alpha = \frac{18}{5}, \quad \beta = -\frac{1}{5}$

$\alpha + \beta = \frac{17}{5} \Rightarrow 15(\alpha + \beta) = 51$

18. Which of the following is logically equivalent ot $(2(p \wedge q) \vee q$

(1) $p \rightarrow q$ (2) $p \rightarrow (p \rightarrow q)$ (3) $p \rightarrow (p \vee q)$ (4) $p \rightarrow (p \wedge q)$

Ans. (3)

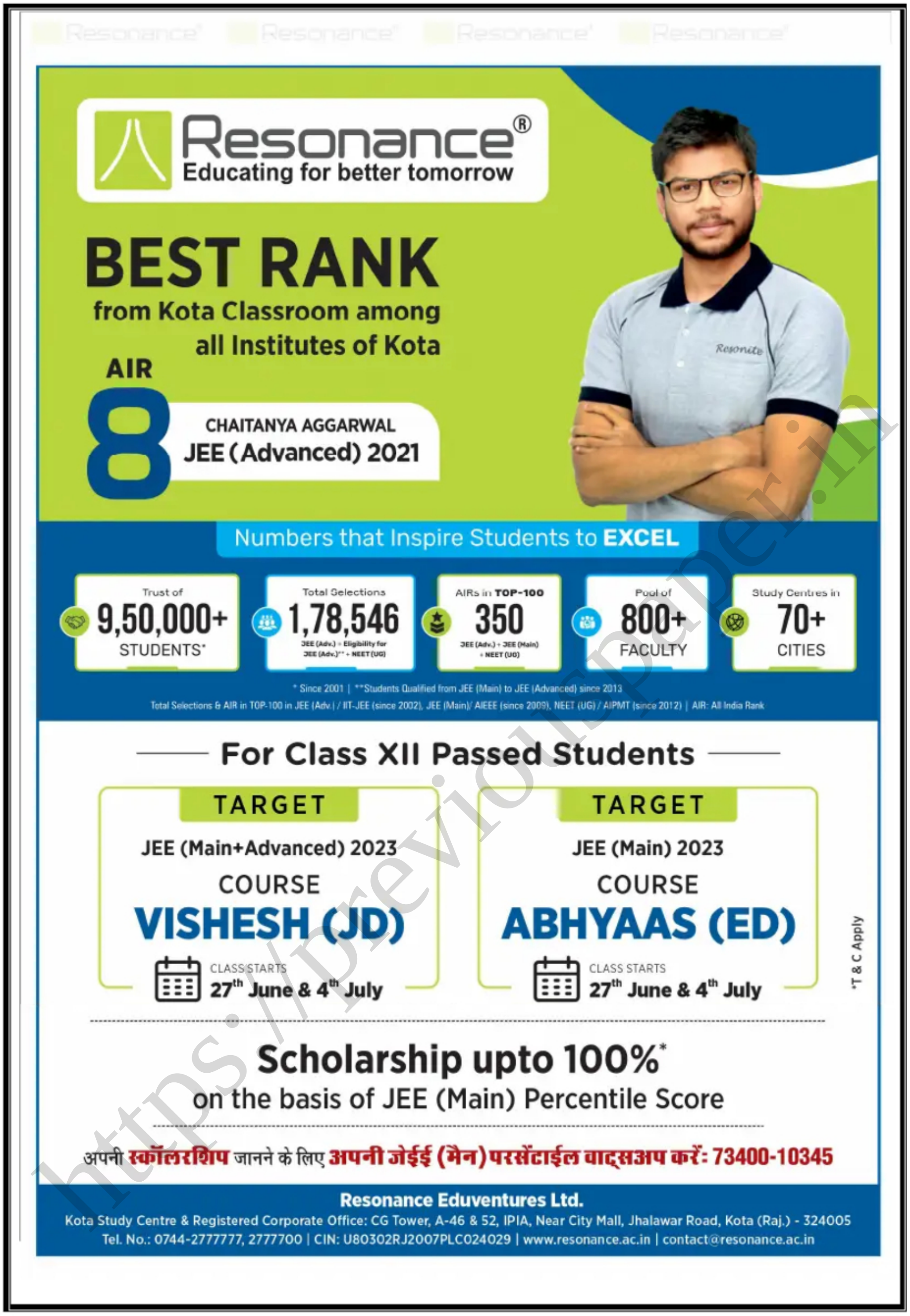
Sol. $\sim(p \wedge q) \vee q = (\sim p \vee \sim q) \vee q$

$= \sim p \vee (\sim q \vee q)$

$= \sim p \vee t = t$

Option (C) $p \rightarrow (p \vee q)$ p $\sim p \vee (p \vee q)$

$= (\sim p \vee p) \vee q = t \vee q = t$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