$-2S_n = 1 + 3 + 3^2 + ... + 3^9 - 10.3^{10}$

$= \frac{1(3^{10}-1)}{3-1} - 10.3^{10}$

$= \frac{1}{2}.3^{10} - \frac{1}{2} - 3^{10}.10$

$-2S_n = -\frac{1}{2} - \frac{19}{2}.3^{10}$

$S_n = \frac{19}{4}3^{10} + \frac{1}{4}$

2. $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right)$ equal to

(1) $-\frac{\pi}{4}$ (2) $-\frac{4\pi}{9}$ (3) $-\frac{\pi}{8}$ (4) $-\frac{3\pi}{8}$

Ans. (3)

Sol. $\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}-1}{\frac{1}{\sqrt{2}}}\right) = \tan^{-1}(1-\sqrt{2}) = -\tan^{-1}(\sqrt{2}-1) = -\frac{\pi}{8}$

3. $2\sin12^\circ - \sin72^\circ$ is equal to

(1) $\sqrt{3}\left(\frac{\sqrt{5}-1}{4}\right)$ (2) $\frac{\sqrt{3}}{4}(1-\sqrt{5})$ (3) $\frac{\sqrt{3}}{4}(1+\sqrt{5})$ (4) None of these

Ans. (2)

Sol. $2(\sin(72^\circ - 60)) - \sin72^\circ$

$2\left(\sin72^\circ.\frac{1}{2} - \cos72^\circ.\frac{\sqrt{3}}{2}\right) - \sin72^\circ$

$-\sqrt{3}\cos(72^\circ)$

$-\sqrt{3}\sin(18^\circ) = -\sqrt{3}\left(\frac{\sqrt{5}-1}{4}\right) = \sqrt{3}\left(\frac{1-\sqrt{5}}{4}\right)$

4. How many three digit numbers in which exactly two digits are same are

Ans. (243)

Sol. 0aa + 00a + aab or abb

${}^9C_1 \times 2$ + ${}^9C_1 \times 1$ + ${}^9C_2 \times {}^2C_1 \times \frac{3!}{2!}$

18 + 9 + 216

243

5. In the expansion of $(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + \ldots\ldots + x^{500}$ then coefficient of x^{101} is

(1) ${}^{501}C_{101}.5^{399}$ (2) ${}^{501}C_{101}5^{400}$ (3) ${}^{501}C_{101}5^{401}$ (4) None of these

Ans. (1)

Sol. $(5+x)^{500}\frac{\left(1-\left(\frac{x}{5+x}\right)^{501}\right)}{1-\frac{x}{5+x}}$

$(5+x)^{501}\frac{\left(1-\frac{x^{501}}{(5+x)^{501}}\right)}{5}$

$\frac{1}{5}\left[(5+x)^{501} - x^{501}\right]$

coefficient of x^{101} is $\frac{1}{5}\times{}^{501}C_{101}5^{400} = {}^{501}C_{101}5^{399}$

6. If z_1 and z_2 are two complex number such that $\bar{z}_1 = i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right) = \pi$ then

(1) $\arg z_1 = \frac{3\pi}{4}$ (2) $\arg z_2 = \frac{-3\pi}{4}$ (3) $\arg z_1 = \frac{\pi}{4}$ (4) $\arg z_2 = \frac{-\pi}{4}$

Ans. **(3)**

Sol. $\arg\left(\frac{z_1}{\bar{z}_2}\right) = \pi \Rightarrow \arg z_1 - \arg \bar{z}_2 = \pi$

$\Rightarrow \arg z_1 + \arg z_2 = \pi$ ....(1)

$\because \bar{z}_1 = i\bar{z}_2 \Rightarrow z_1 = -iz_2$

$\Rightarrow \arg z_1 = \arg(-iz_2)$

$\Rightarrow \arg z_1 = \arg(-i) + \arg z_2$

$\Rightarrow \arg z_1 - \arg z_2 = -\frac{\pi}{2}$ ....(2)

from (1) and (2) $\arg z_1 = \frac{\pi}{4}$ and $\arg z_2 = \frac{3\pi}{4}$

7. $\frac{x^2}{4} + \frac{y^2}{2} = 1$ and line y = x + 1 intersect at P & Q. A circle is described by assuming PQ as a diameter with radius r then the value of $(3r)^2$ is

(1) 20 (2) 22 (3) 25 (4) 30

Ans. **(1)**

Sol. $\frac{x^2}{4} + \frac{(x+1)^2}{2} = 1$

$x^2 + 2(x + 1)^2 = 4$

$3x^2 + 4x - 2 = 0$

$(x_1 - x_2) = \sqrt{(x_1 + x_2)^2 - 4x_1x_2}$

$= \sqrt{\frac{16}{9} + \frac{8}{3}}$

$= = \sqrt{\frac{40}{9}}$

also $(y_1 - y_2) = |x_1 - x_2| = \sqrt{\frac{40}{9}}$

radius $= \frac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \frac{1}{2}\sqrt{2 \times \frac{40}{9}}$

$= \frac{1}{2} \frac{\sqrt{80}}{3}$

$r = \frac{\sqrt{80}}{6}$

$(3r)^2 = 9r^2 = 9 \times \frac{80}{36} = 20$

8. A biased dice whose faces are 2, 4, 8, 16, 32, 32 is rolled thrice. If probability of getting number n on dice in one trial is 1/n then probability of getting sum 48 is

(1) $\frac{3}{2^{12}}$ (2) $\frac{7}{2^{11}}$ (3) $\frac{13}{2^{12}}$ (4) $\frac{9}{2^{12}}$

Ans. **(2)**

Sol. Given $P(n) = \frac{1}{n}$

Favourable cases (16, 16, 16), (32, 8, 8), (8, 32, 8), (8, 8, 32)

$\text{Prob} = \left(\frac{1}{16} \times \frac{1}{16} \times \frac{1}{16}\right) + 3 \times \left(\frac{1}{8} \times \frac{1}{8} \times \frac{1}{32}\right) = \frac{1}{2^{12}} + \frac{3}{2^{11}} = \frac{7}{2^{12}}$

9. If equation $ax^2 - 2bx + 15 = 0$ has repeated root α and equation $x^2 - 2bx + 21 = 0$ has root α and β. then the value of $\alpha^2 + \beta^2$

(1) 57 (2) 58 (3) 59 (4) 60

Ans. **(2)**

Sol. Equation

$ax^2 - 2bx + 15 = 0$ with roots α, α

Sum of root $\alpha + \alpha = \frac{2b}{a} \Rightarrow \alpha = b/a$

Product of root $\alpha \cdot \alpha = \frac{15}{a} \Rightarrow a = \frac{15}{\alpha^2}$

since roots are repeated $\Rightarrow$ D = 0

$\Rightarrow 4b^2 - 4a.15 = \Rightarrow b^2 = 15a = \frac{15^2}{\alpha^2} \Rightarrow b = \pm \frac{15}{\alpha}$

Equation

$x^2 - 2bx + 21 = 0$ (roots α, β)

Sum of root $\alpha + \beta = 2b$,

Product of root $\alpha\beta = 21$

$\alpha + \frac{21}{\alpha} = 2\left(\frac{15}{\alpha}\right) \Rightarrow \alpha^2 = 9 \Rightarrow \alpha = \pm 3$

$\Rightarrow \beta = \pm 7$

The value of

$\alpha^2 + \beta^2 = 9 + 49 = 58$

10. $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$ & y(e) = e/3 then the value y(1) is

(1) 2/3 (2) 3/2 (3) 1/3 (4) None

Ans. **(1)**

Sol. $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{xy} + \frac{3}{2x^2} = 0$

Let $\frac{1}{y} = t$

Difference of

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$-\frac{dt}{dx} - \frac{1}{x}t + \frac{3}{2x^2} = 0$

$\frac{dt}{dx} + \frac{1}{x}t = \frac{3}{2x^2}$

I.F. $= e^{\int \frac{1}{x}dx} = x$

$x.t. = \int x.\frac{3}{2x^2}dx$

$x.t. = \frac{3}{2}\ell nx + c$

$\frac{x}{y} = \frac{3}{2}\ell nx + c$

x = e, y = e/3

$\frac{e}{e/3} = \frac{3}{2}.1 + c$

$\Rightarrow c = \frac{3}{2}$

$\frac{x}{y} = \frac{3}{2}\ell nx + \frac{3}{2}$

$\Rightarrow x = 1$

$\Rightarrow \frac{1}{y} = \frac{3}{2}$

y = 2/3

11. Locus of centre of a circle which touches the line $x - y = 0$ and y – axis, is

(1) $x^2 + 2xy - y^2 = 0$ (2) $x^2 - 2xy + y^2 = 0$

(3) $2x + y^2 + xy = 0$ (4) $x^2 + 4xy = 0$

Ans. (1)

Sol. Let centre of circle be C(h, k)

$\because$ circle touches y – axis $\Rightarrow$ radius = |h|

$\because$ circle touches $x - y = 0$

$\Rightarrow \dfrac{|h-k|}{\sqrt{2}} = |h|$

$\Rightarrow (h-k)^2 = 2h^2$

$\Rightarrow h^2 + k^2 - 2hk = 2h^2$

$\Rightarrow h^2 + 2hk - k^2 = 0$

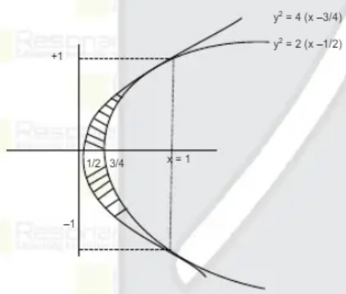
locus of (h, k) $\rightarrow x^2 + 2xy - y^2 = 0$

12. The area bounded by $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

(1) 1/3 (2) 1/4 (3) 1/5 (4) 1/6

Ans. (1)

Sol.



When $y^2 = 2x - 1$

$y^2 = 4x - 3$

$4x - 3 = 2x - 1$

$x = 1,\ y = \pm 1$

Point of intersection are (1,1) , (1, –1)

Required area = $\int_{-1}^{1}(x_1 - x_2)\,dy = \int_{-1}^{1}\left(\dfrac{y^2+3}{4} - \dfrac{y^2+1}{2}\right)dy = \int_{-1}^{1}\left(\dfrac{-y^2+1}{4}\right)dy$

$= \dfrac{1}{4}\left(\dfrac{-y^3}{3} + y\right)_{-1}^{1} = \dfrac{1}{4}\left(\left(1-\dfrac{1}{3}\right) - \left(\dfrac{1}{3} - 1\right)\right) = \dfrac{1}{3}$

13. If solution set of $|x + 1| < 2$ is A and $|x - 1| \geq 2$ is B then which of following is incorrect

(1) $A \cap B = (-3, -1]$ (2) $A \cup B = (-\infty, 1) \cup [3, \infty)$

(3) $A - B = A \cap B' = (-1, 1)$ (4) $B - A = A' \cap B = (-\infty, -2] \cup [2, \infty)$

Ans. (4)

Sol. $|x + 1| < 2 \Rightarrow -2 < x + 1 < 2 \Rightarrow x \in (-3, 1) = A$

$|x - 1| \geq 2 \Rightarrow x - 1 \leq -2 \cup x - 1 \geq 2$

$\Rightarrow x \leq -1 \cup x \geq 3 \Rightarrow x \in (-\infty, -1] \cup [3, \infty)$

$A = (-3, 1)$ $B = (-\infty, -1] \cup [3, \infty)$

$A' = (-\infty, -3] \cup [1, \infty)$ $B' = (-1, 3)$

$A - B = A \cap B' = (-1, 1)$

B – A = A' ∩ B = (–∞, –3] ∪ [3, ∞)
A ∪ B = (–∞, 1) ∪ [3, ∞)
A ∩ B = (–3, –1]

14. If line y = kx + 4 is tangent to parabola $y = x - x^2$ at point 'P' then find slope of 'OP' where 'O' is vertex of parabola

(1) $-\frac{3}{2}$ (2) $-\frac{5}{4}$ (3) $-\frac{5}{2}$ (4) $\frac{3}{2}$

Ans. (1)

Sol. $y = kx + 4$
$y = x - x^2$
$kx + 4 = x - x^2$ ....(1)
$x^2 + (k - 1)x + 4 = 0$
$(k - 1)^2 - 4.4 = 0$
$k - 1 = \pm 4$
if k = 5
now put the value of k = 5
$5x + 4 = x - x^2$
$x^2 + 4x + 4 = 0$
$(x + 2)^2 = 0$
x = –2
y = – 6
if k = –3
now put the value of k = –3 in equation (1)
$-3x + 4 = x - x^2$
$x^2 - 4x + 4 = 0$
x = 2 y = – 2
then the point of P is (2,–2) and (–2,–6)

and vertex of parabola 'O' = $y - \frac{1}{4} = -\frac{1}{4} + x - x^2$

$y - \frac{1}{4} = -\left(x - \frac{1}{2}\right)^2$

point P is (2,–2)

slope of OP = $\frac{-2 - \frac{1}{4}}{2 - \frac{1}{2}} = -\frac{9}{3 \times 2} = -\frac{3}{2}$

point P is (–2,–6) slope of OP $\frac{-6 - \frac{1}{4}}{-2 - \frac{1}{2}} = \frac{5}{2}$

15. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2(nx)}{\sin x}$ dx then.

(1) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b4}$ are in A.P with common difference 2

(2) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b4}$ are in A.P with common difference –2

(3) $b_3 - b_2$, $b_4 - b_3$, $b_5 - b_4$ are in A.P with common difference 2

(4) $b_3 - b_2$, $b_4 - b_3$, $b_5 - b_4$ are in A.P with common difference – 2

Ans. (2)

Sol. $b_n - b_{n-1} = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx - \cos^2(n-1)x}{\sin x}$

$= \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x \sin(-x)}{\sin x}$

$= -\int_0^{\frac{\pi}{2}} \sin(2n-1)x \; dx$

$= \frac{1}{2n-1} \left[\cos(2n-1)x\right]_0^{\frac{\pi}{2}} = -\frac{1}{2n-1}$

$b_n - b_{n-1} = -\frac{1}{2n-1}$

$$\frac{1}{b_n - b_{n-1}} = -(2n-1)$$

$$\frac{1}{b_3 - b_2} = -5, \quad \frac{1}{b_4 - b_3} = -7, \quad \frac{1}{b_5 - b_4} = -9$$

common difference = – 2

16. Negation of $(p \wedge \sim q) \rightarrow (\sim p \vee q)$ is

(1) $\sim(p \rightarrow q)$ (2) $(\sim p \rightarrow q)$ (3) $(p \rightarrow q)$ (4) $p \rightarrow \sim q$

Ans. **(1)**

Sol. $(p \wedge \sim q) \wedge (p \vee \sim q)$

$= (p \wedge \sim q)$

$= \sim(p \rightarrow q)$

17. For the first n natural numbers, where n is odd, the mean deviation from mean is $5\frac{(n+1)}{n}$ then the value of n is

Ans. **21.00**

Sol. $\bar{x} = \frac{1+2+3+.....+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$ (which is middle term)

$$\text{M.D.} = \frac{\Sigma |x_i - \bar{x}|}{n} = \frac{5(n+1)}{n}$$

$$\Rightarrow \frac{2\left(1+2+3+....+\frac{(n-1)}{2}\right)}{n} = \frac{5(n+1)}{n}$$

$$\Rightarrow \frac{2.\frac{\left(\frac{(n-1)}{2}\right)\left(\frac{(n-1)}{2}+1\right)}{2}}{n} = \frac{5(n+1)}{n}$$

$$\Rightarrow \frac{(n-1)(n+1)}{4n} = \frac{5(n+1)}{n}$$

n = 21

18. If m and n are respectively the local maxima & local minima of $f(x) = |(x-1)(x^2+2x-3)|.(x+3)$ then the value of m + n is

Ans. **(00.00)**

Sol. $f(x) = |(x-1)^2(x+3)|(x+3)$

$$f(x) = \begin{cases} (x-1)^2(x+3)^2, & x \geq -3 \\ -(x-1)^2(x+3)^2, & x < -3 \end{cases}$$

$$f(x) = \begin{cases} (x^2+2x-3)^2, & x \geq -3 \\ -(x^2+2x-3)^2, & x < -3 \end{cases}$$

$$f'(x) = \begin{cases} 2(x^2+2x-3)(2x+2), & x \geq -3 \\ -2(x^2+2x-3)(2x+2), & x < -3 \end{cases}$$

$$f'(x) = \begin{cases} 4(x+3)(x-1)(x+1), & x \geq -3 \\ -4(x+3)(x-1)(x+1), & x < -3 \end{cases}$$

Local maxima $\rightarrow$ m = –1

Local Minima $\rightarrow$ n = 1

m + n = 0