$$S = 100 - \left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \text{......100 term}\right)$$

$$S = 100 - \frac{2}{3}\left(\frac{1-\left(\frac{2}{9}\right)^{100}}{1-\frac{2}{3}}\right)$$

$$S = 100 - 2\left(1-\left(\frac{2}{3}\right)^{100}\right)$$

$$S = 98 + 2\left(\frac{2}{3}\right)^{100}$$

$$0 < 2 \times \left(\frac{2}{3}\right)^{100} < 1$$

[S] = 98

2. If $a_n = 19^n - 12^n$, then the value of $\frac{31a_9 - a_{10}}{57a_8}$ is

(1) 4 (2) 5 (3) 7 (4) 8

Ans. (1)

Sol. $\Rightarrow \frac{31(19^9 - 12^9) - (19^{10} - 12^{10})}{57(19^8 - 12^8)}$

$\Rightarrow \frac{19^9(12) - 12^9(19)}{57(19^8 - 12^8)}$

$\Rightarrow \frac{19 \times 12(19^8 - 12^8)}{57(19^8 - 12^8)}$

$\Rightarrow 4$

3. If $f(x) = \left\{\left(2\left(1-\frac{x}{2}\right)\right)^{25}(2+x)^{25}\right\}^{1/50}$ and g(x) = f(f(f((x))) + f(f(x)) then find then [g(1)] is ______

(where [x] is greatest integer less than or equal to x).

Ans. (2)

Sol. g(x) = f(f(f((x))) + f(f(x))
g(1) = f(f(f((1))) + f(f(1)) ....(1)
$f(1) = (\sqrt{3})$
$f(f(1)) = f(\sqrt{3}) = ((2 - x)(2 + x))^{1/2}$
$= (4 - 3)^{1/2} = 1$
$f(f(f(1))) = f(1) = \sqrt{3}$
$\Rightarrow [g(1)] = [\sqrt{3} + 1] = 2$

4. The value of integral $I = \int_0^{\pi} \frac{e^{\cos x}\sin x}{\left(e^{\cos x} + \frac{1}{e^{\cos x}}\right)(1+\cos^2 x)}dx$ is

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

Ans. (3)

Sol. $\int_0^{\pi} \frac{e^{\cos x}\sin x}{\left(e^{\cos x} + \frac{1}{e^{\cos x}}\right)(1+\cos^2 x)}dx$

By using P – VI

$$\int_0^{\frac{\pi}{2}}\left\{\frac{e^{\cos x}\sin x}{\left(e^{\cos x}+\frac{1}{e^{\cos x}}\right)(1+\cos^2 x)}+\frac{\frac{1}{e^{\cos x}}\sin x}{\left(\frac{1}{\left(e^{\cos x}+e^{\cos x}\right)}\right)}\right\}dx$$

$$\int_0^{\pi/2}\frac{\sin x}{1+\cos^2 x}dx$$

cosx = t

– sinxdx = dt

x	0	$\pi/2$
t	1	0

$$=\int_1^0\frac{-dt}{1+t^2}=\left(\cot^{-1}t\right)_1^0=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

5. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 4$, $|\vec{b}| = 3$ and angle between $\vec{a}$ and $\vec{b}$ lies in $\left(\frac{\pi}{4},\frac{\pi}{3}\right)$ then the value of $\left((\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right)^2+4(\vec{a}.\vec{b})^2$ is

(1) 576 (2) 1152 (3) 289 (4) 432

Ans. (1)

Sol. Given $|\vec{a}| = 4$, $|\vec{b}| = 3$, $\vec{a}\wedge\vec{b}\in\left(\frac{\pi}{4},\frac{\pi}{3}\right)$

$\left((\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right)^2+4(\vec{a}.\vec{b})^2$

$\left(-\vec{a}\times\vec{b}+\vec{b}\times\vec{a}\right)^2+4(\vec{a}.\vec{b})^2$

$4\left(\vec{a}\times\vec{b}\right)^2+4(\vec{a}.\vec{b})^2$

$= 4\,|\vec{a}|^2\,|\vec{b}|^2$

$= 4 \times 16 \times 9 = 64 \times 9 = 576$

6. In a ΔABC, if $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}$ then $\frac{R}{r}$ = ?

Ans. **02.50**

Sol. a + b = 7 k

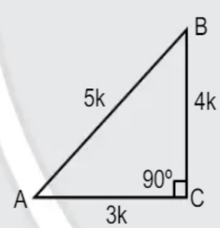
b + c = 8 k

a + c = 9 k

2(a + b + c) = 24 k

a + b + c = 12 k

a = 4k, b = 3k, c = 5k (triangle is right angle at C)



$2R=\frac{C}{\sin C}=\frac{5k}{\sin(90^\circ)}\Rightarrow R=\frac{5k}{2}$

$r=\frac{\Delta}{s}=\frac{\frac{1}{2}(3k)(4k)}{6k}=k$

$\frac{R}{r}=\frac{5k/2}{k}=\frac{5}{2}$ = 2.5

7. If f(x) = $x^3 + x - 5$ and g(x) be another function such that g(f(x)) = x then find the value of g′(63)

(1) $\frac{1}{50}$ (2) $\frac{1}{49}$ (3) $\frac{1}{48}$ (4) $\frac{1}{47}$

Ans. (2)

Sol. $g'(f(x))=\frac{1}{f'(x)}$ ....(1)

to find g′(63) put f(x) = 63

$x^3 + x - 5 = 63$

$x(x^2 + 1) = 68 \Rightarrow x = 4$

Now $f'(x) = 3x^2 + 1 \Rightarrow f'(4) = 49$

then the value of $g'(63) = \frac{1}{49}$

8. If A and B are unit vectors and θ is acute angle between them, choose the correct option

(1) $|\vec{A}-\vec{B}| = |\vec{A}+\vec{B}|\tan\frac{\theta}{2}$ (2) $|\vec{A}-\vec{B}| = |\vec{A}+\vec{B}|\cos\frac{\theta}{2}$

(3) $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|\cos\frac{\theta}{2}$ (4) $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|\tan\frac{\theta}{2}$

Ans. **(1)**

Sol. $|\vec{A}+\vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta$

$= 1 + 1 + 2\cos\theta$

$= 2(1 + \cos\theta)$

$= 4\cos^2\frac{\theta}{2}$ ....(1)

and

$|\vec{A}-\vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta$

$= 1 + 1 - 2\cos\theta$

$= 2(1 - \cos\theta)$

$= 4\sin^2\frac{\theta}{2}$ ....(2)

by eq. (1) and (2)

$\Rightarrow \frac{|\vec{A}-\vec{B}|}{|\vec{A}+\vec{B}|} = \tan\frac{\theta}{2} \Rightarrow |\vec{A}-\vec{B}| = |\vec{A}+\vec{B}|\tan\frac{\theta}{2}$

9. $\frac{1}{2^1.3^{10}} + \frac{1}{2^2.3^9} + \ldots\ldots + \frac{1}{2^9.3^2} + \frac{1}{2^{10}3^1} = \frac{k}{2^{10}.3^{10}}$ then find remainder when k is divided by 6 is

(1) 2 (2) 3 (3) 4 (4) 5

Ans. **(4)**

Sol. $\frac{1}{2^1.3^{10}} + \frac{1}{2^2.3^9} + \ldots\ldots + \frac{1}{2^9.3^2} + \frac{1}{2^{10}3^1} = \frac{k}{2^{10}.3^{10}}$

$\Rightarrow k = 2^9 + 2^8.3 + 2^7.3^2 + \ldots\ldots\ldots\ldots + 2.3^8 + 3^9 = 3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5)$

$= 211 \times 275$

$= (210 + 1)(270 + 5)$

$= (6\lambda + 1)(6\mu + 5)$

Remainder = 5

10. If f(x) is a function which satisfies $f(x + y) = 2f(x).f(y)$, $f(1) = 2$ and $\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}(2^{20}-1)$, $\alpha \in W$ then possible value of α is

Ans. **04.00**

Sol. $\because f(x + y) = 2f(x).f(y)$

Let $f(x) = 2^{2x-1}$

Now $\sum_{k=1}^{10} f(\alpha+k) = \sum_{k=1}^{10} 2^{2(\alpha+k)-1} = 2^{2\alpha-1}.\sum_{k=1}^{10} 2^{2k}$

$= 2^{2\alpha-1}.2^2\frac{\left[(2^2)^{10}-1\right]}{2^2-1} = \frac{2^{2\alpha+1}\left[2^{20}-1\right]}{3}$

$2^{2\alpha+1} = 512 \Rightarrow 2\alpha + 1 = 9$

$\alpha = 4$

11. If f(x) be a polynomial in x such that $f(x) + f'(x) + f''(x) = x^5 + 64$ then value of $\lim_{x\to1}\frac{f(x)}{x-1}$ is

(1) 15 (2) –14 (3) 14 (4) –15

Ans. **(4)**

Sol. As $f(x) + f'(x) + f''(x) = x^5 + 64$

$\Rightarrow$ f(x) is a polynomial in x with degree 5

Let $f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$

$\Rightarrow f'(x) = 5x^4 + 4ax^3 + 3bx^2 + 2cx + d$

& $f''(x) = 20x^3 + 12ax^2 + 6bx + 2c$

Since $f(x) + f'(x) + f''(x) = x^5 + 64$

$\Rightarrow a + 5 = 0,\ b + 4a + 20 = 0\ ;\ c + 3b + 12a = 0,\quad d + 2c + 6b = 0\ \&\ e + d + 2c = 64$

$\Rightarrow a = -5,\ b = 0,\ \ c = 60,\ \ d = -120,\ e = 64$

$\Rightarrow f(x) = x^5 - 5x^4 + 60x^2 - 120x + 64$

$\Rightarrow f(x) = (x-1)(x^4 - 4x^3 - 4x^2 + 56x - 64)$

$\Rightarrow \frac{f(x)}{x-1} = x^4 - 4x^3 - 4x^2 + 56x - 64$

$\lim_{x\to1}\frac{f(x)}{x-1} = 1 - 4 - 4 + 56 - 64 = -15$

12. If m_1 and m_2 are slope of the common tangents of curves $x^2 + y^2 = 2$ and $y^2 = x$ then the value of $8.|m_1.m_2|$ is

(1) $3\sqrt{2} - 3$ (2) $3\sqrt{2} - 4$ (3) $3\sqrt{2} - 5$ (4) $3\sqrt{5} - 4$

Ans. **(2)**

Sol. Let equation of tangent $y = mx + \frac{1}{4m}$

then is also tangent to curve

$$\left|\frac{0 - 0 + \frac{1}{4m}}{\sqrt{m^2+1}}\right| = \sqrt{2}$$

$$\frac{1}{16m^2} = 2(m^2 + 1)$$

$32m^4 + 32m^2 - 1 = 0$

$$m^2 = \frac{-32 + \sqrt{(32)^2 + 4\times32}}{64}$$

then the value of $8.|m_1.m_2| = 8m^2$

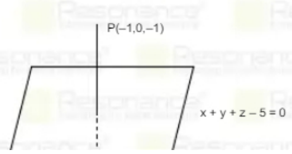
$$= 8 \times \left(\frac{-32 + \sqrt{(32)^2 + 4\times32}}{64}\right)$$

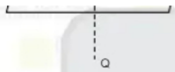
$= -4 + \sqrt{18} = 3\sqrt{2} - 4$

13. If Q is the mirror image of P(–1, 0 –1) with respect to plane x + y + z = 5 and a line parallel to PQ passes through A(1, –1,–1) intersect given plane at a point R, then the value of QR^2 is

Ans. **(21.00)**

Sol.





Coordinate of Q

$\frac{x+1}{1}=\frac{y-0}{1}=\frac{z+1}{1}=-2\left(\frac{-1+0-1-5}{3}\right)=\frac{14}{3}$

$Q\left(\frac{11}{3},\frac{14}{3},\frac{11}{3}\right)$

Direction ratio of PQ = $\frac{14}{3},\frac{14}{3},\frac{14}{3}$

equation of line passes through A(1, –1, –1) and parallel to PQ is

$\frac{x-1}{\frac{14}{3}}=\frac{y+1}{\frac{14}{3}}=\frac{z+1}{\frac{14}{3}}$

$\Rightarrow \frac{x-1}{1}=\frac{y+1}{1}=\frac{z+1}{1}=\lambda$

$R(\lambda + 1, \lambda - 1, \lambda - 1)$

R lies on given plane

so, $(\lambda + 1) + (\lambda - 1) + (\lambda - 1) = 5$

$\Rightarrow \lambda = \frac{6}{3}=2$

then the point R(3,1, 1)

$QR^2 = \left(\frac{11}{3}-3\right)^2+\left(\frac{14}{3}-1\right)^2+\left(\frac{11}{3}-1\right)^2 = \frac{4}{9}+\frac{121}{9}+\frac{64}{9}=\frac{189}{9}=21$

14. Number of all three digits odd integers whose sum of digit is a multiple of 7, is

Ans. **(57.00)**

Sol. Largest digit in number = 9

Sum of three digit maximum can be = 27

Hence sum of digit can be = 7, 14, 21

Now $\boxed{a_1}\boxed{a_2}\boxed{a_3}$ $a_1 + a_2 + a_3 = 7, 14, 21$

C-1 $a_1 + a_2 + a_3 = 7$ a_3 = odd < C

$a_3 = 1$ $(a_1, a_2) \equiv (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)$ Total number = 5

$a_3 = 3$ $(a_1, a_2) \equiv (1, 3), (2, 2), (3, 1)$ Total number = 3

$a_3 = 5$ $(a_1, a_2) \equiv (1, 1)$ Total number = 1

C-2 $a_1 + a_2 + a_3 = 14$

$a_3 = 1$ $(a_1, a_2) \equiv (4, 9), (5, 8)$,.......(9, 4) Total number = 6

$a_3 = 3$ $(a_1, a_2) \equiv (2, 9), (3, 8)$,.......(9, 2) Total number = 8

$a_3 = 5$ $(a_1, a_2) \equiv (1, 8)$,.......(8, 1) Total number = 8

$a_3 = 7$ $(a_1, a_2) \equiv (1, 6)$,.......(6, 1) Total number = 6

$a_3 = 9$ $(a_1, a_2) \equiv (1, 4)$,.......(4, 1) Total number = 4

C-3 $a_1 + a_2 + a_3 = 21$

$a_3 = 3$ $(a_1, a_2) \equiv (9, 9)$ Total number = 1

$a_3 = 5$ $(a_1, a_2) \equiv (7, 9), (8, 8), (9, 7)$ Total number = 3

$a_3 = 7$ $(a_1, a_2) \equiv (5, 9)$,.......(9, 5) Total number = 5

$a_3 = 9$ $(a_1, a_2) \equiv (3, 9)$,.......(9, 3) Total number = 7

Total = 57

15. If $z_1 = 4 + 3i$, $z_2 = 3 + 4i$ and $z_3 = 5i$ lie on a circle and z lie on same circle such that line joining z and z_1 is perpendicular to line joining z_2 and z_3 then arg(z) equal to

(1) $\tan^{-1}\frac{24}{7}$ (2) $-\tan^{-1}\frac{24}{7}$ (3) $\pi - \tan^{-1}\frac{24}{7}$ (4) $-\pi + \tan^{-1}\frac{24}{7}$

Ans. **(2)**

Sol. Equation of circle passes through $z_1(4 + 3i)$, $z_2(3 + 4i)$ and z_3 (5i) is $x^2 + y^2 = 25$

slope of $z_2 z_3$ = $\frac{5-4}{0-3}=-\frac{1}{3}$

slope of zz_1 = 3

Equation of zz_1 is $y - 3 = 3(x - 4)$

$y = 3x - 9$

Now $x^2 + (3x - 9)^2 = 25$

$10x^2 - 54x + 81 - 25 = 0$

$10x^2 - 54x + 56 = 0$

$(x-4)(5x-7)=0$

$x=4$ or $\frac{7}{5}$

at $x=4$; $y=3$

$x=\frac{7}{5}$, $y=-\frac{24}{5}$

So $z\left(\frac{7}{5},-\frac{24}{5}\right)$

So $\arg(z) = -\tan^{-1}\frac{24}{7}$

16. Let $A=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$ and $M=\sum_{k=1}^{10}A^{2K}$; $N=\sum_{k=1}^{10}A^{2K-1}$ then MN^2 is

(1) symmetric matrix
(2) skew symmetric matrix
(3) Identify matrix
(4) singular matrix

Ans. (1)

Sol. $A^2=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}=\begin{bmatrix}-4 & 0\\ 0 & -4\end{bmatrix}=-4I.$ (symmetric)

& $A^3=-4A$ (skew symmetric)

$\Rightarrow M=\sum_{k=1}^{10}A^{2K} = [(-4)+(-4)^2+(-4)^3+....+(-4)^{10}]\ I$

$=-4\lambda I$ is symmetric

$\Rightarrow N=\sum_{k=1}^{10}A^{2K-1} = A[1+(-4)+(-4)^3+....+(-4)^9]\ I$

$=\lambda A$ is skew symmetric

Where $\lambda=\{1+(-4)+(-4)^3+....+(-4)^9\}$

Now $MN=-4\lambda^2 A=NM$

$\Rightarrow$ $MN^2=(MN)N=(NM)N=N(MN)=N(NM)=N^2M$

Hence $(MN^2)^T=(N^2)^T M^T=(N^T)^2 M^T=(-N)^2 M=N^2M$

$\Rightarrow MN^2$ is symmetric matrix

17. Let E_1 and E_2 be two events such that $P(E_1/E_2)=\frac{1}{2}$, $P(E_2/E_1)=\frac{3}{4}$

(1) $P(E_1\cap E_2)=P(E_1)P(E_2)$
(2) $P(\overline{E}_1\cap\overline{E}_2)=P(\overline{E}_1)P(E_2)$
(3) $P(\overline{E}_1\cap E_2)=P(E_1)P(E_2)$
(4) $P(E_1\cap\overline{E}_2)=P(E_1)P(E_2)$

Ans. (4)

Sol. $P(E_1\cap E_2)=\frac{1}{8}=P(E_1)\ P\left(\frac{E_2}{E_1}\right)=P(E_2)\ P\left(\frac{E_1}{E_2}\right)$

So $\frac{1}{8}=P(E_1)\times\frac{3}{4}$

$P(E_1)=\frac{1}{6}$

And $\frac{1}{8}=P(E_2)\times\frac{1}{2}$

$P(E_2)=\frac{1}{4}$

Now option

(1) $P(E_1)\ P(E_2)=\frac{1}{6}\times\frac{1}{4}=\frac{1}{24}\neq P(E_1\cap E_2)$

(2) $P(\overline{E}_1)P(\overline{E}_2)=\frac{5}{6}\times\frac{3}{4}=\frac{15}{24}$

$P(\overline{E}_1\cap\overline{E}_2)=P\left(\overline{E_1\cup E_2}\right)=1-P(E_1\cup E_2)$

$=1-[P(E_1)+P(E_2)-P(E_1\cap E_2)]=1-\left(\frac{1}{6}+\frac{1}{4}-\frac{1}{8}\right)=1-\frac{7}{24}=\frac{17}{24}$

$P(\overline{E}\cap\overline{E}_2)\neq P(\overline{E}_1)P(\overline{E}_2)$

(3) $P(\overline{E}_1\cap E_2)=P(E_2)-P(E_1\cap E_2)$

$=\frac{1}{4}-\frac{1}{8}=\frac{1}{8}\neq P(E_1)\ P(E_2)$

(4) $P(E_1\cap\overline{E}_2)=P(E_1)-P(E_1\cap E_2)$

$$= \frac{1}{6} - \frac{1}{8} = \frac{1}{24} = P(E_1)\,P(E_2)$$