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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 26 June, 2022 (SHIFT-2) | TIME : (3.00 a.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. The value of $\lim_{x\to 0}\dfrac{\cos(\sin x)-\cos x}{x^4}$ is

(1) $\dfrac{1}{2}$ (2) $\dfrac{1}{6}$ (3) $\dfrac{1}{12}$ (4) $\dfrac{1}{3}$

Ans. (2)

$-2\sin\left(\dfrac{\sin x+x}{2}\right)\sin\left(\dfrac{\sin x-x}{2}\right)$

Sol.. $\lim_{x\to 0} \frac{\ldots 2 \,)\,(\,\ldots 2\,)}{x^4}$

$\Rightarrow \lim_{x\to 0} \frac{-2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{\sin x - x}{2}\right)}{x^4}$

$\Rightarrow -\frac{1}{2} \lim_{x\to 0} \frac{\left(\frac{\sin x}{x}+1\right)(\sin x - x)}{x^3}$

$\Rightarrow -\frac{1}{2}(1+1) \lim_{x\to 0} \frac{\left[\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots\ldots\right) - x\right]}{x^3}$

$\Rightarrow \frac{-1}{2}(2)\left(-\frac{1}{3!}\right) = \frac{1}{6}$

2. The value of 16(sin20º) (sin40º) (sin80º) is

(1) $2\sqrt{3}$ (2) $2\sqrt{5}$ (3) $-2\sqrt{5}$ (4) $-2\sqrt{3}$

Ans. **(1)**

Sol. $\because$ sinx· sin (60º–x)·sin (60º+x) = $\frac{\sin 3x}{4}$

$\Rightarrow$ 16(sin20º) (sin40º) (sin80º)

= 16(sin20º) (sin(60º – 20º)) (sin60º + 20º)

$= 16 \times \frac{\sin 60^\circ}{4}$

$= 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$

3. Let y(x) represents the solution of differential equation

$x\frac{dy}{dx} + 2y = xe^x$ and z(x) = x^2 y(x) – e^x, and y(1) = 0, then the maximum value of z(x) is

(1) $\frac{4}{e} - e$ (2) $\frac{4}{e^2} - e$ (3) $\frac{2}{e} - e$ (4) $\frac{2}{e^2} - e$

Ans. (1)

Sol. $x\frac{dy}{dx} + 2y = xe^x$

$\frac{dy}{dx} + \frac{2}{x}.y = e^x$

I.F. = $e^{2\int \frac{1}{x}dx} = x^2$

final solution is

y.x^2 = $\int x^2.e^x\,dx$

$yx^2 = x^2e^x - 2xe^x + 2e^x + k$

given y(1) = 0

x = 1, y = 0 given

0 = e – 2e + 2e + k $\Rightarrow$ k = – e

$y.x^2 - e^x = x^2e^x - 2xe^x + e^x - e$

z(x) = $(x-1)^2 e^x - e$

z'(x) = $2(x-1)e^x + (x-1)^2 e^x$

for maximum or minimum z′(x) = 0

$(x-1)e^x(2+(x-1)) = 0$

$(x-1)(x+1)e^x = 0$

$z(x)\big|_{max.} = 4e^{-1} - e = \frac{4}{e} - e$

4. If $z^2 + z + 1 = 0$, ($z \in C$) then the value of $\left|\sum_{n=1}^{15}\left(z^n + \frac{(-1)^n}{z^n}\right)^2\right|$ is equal to

Ans. **(2)**

Sol. z = w, w^2

$\left(z^n + \frac{(-1)^n}{z^n}\right)^2 = z^{2n} + \frac{1}{z^{2n}} + (-1)^n.2$

exp. $=\left|\sum_{n=1}^{15}\left(z^n+\frac{(-1)^n}{z^n}\right)^2\right| = \left|\sum_{n=1}^{15} z^{2n} + 2\sum_{n=1}^{15}(-1)^n + \sum\frac{1}{z^{2n}}\right|$

$= 0 + 0 + 2(1) = 2$

5. Let p, q ∈ R such that $p + q = 3$ and $p^4 + q^4 = 369$ then the value of $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is

(1) 5 (2) $\frac{1}{4}$ (3) 4 (4) $\frac{1}{5}$

Ans. (3)

Sol. $p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$

$\Rightarrow \quad 369 = \left((p+q)^2 - 2pq\right)^2 - 2(pq)^2$

$\Rightarrow \quad 369 = (9 - 2x)^2 - 2x^2$ (where x = pq)

$\Rightarrow \quad 369 = 81 - 36x + 4x^2 - 2x^2$

$\Rightarrow \quad 2x^2 - 36x - 288 = 0$

$\Rightarrow \quad x^2 - 18x - 144 = 0$

$\Rightarrow \quad (x - 24)(x + 6) = 0$

$\Rightarrow \quad x = 24, -6$

$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2} = \left(\frac{p+q}{pq}\right)^{-2} = \left(\frac{pq}{p+q}\right)^2 = \left(\frac{-6}{3}\right)^2 = 4$

6. If the common tangent of $\frac{x^2}{16}+\frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$ is $y = mx + c$ then the value of $12m^2$ is

(1) 12 (2) 10 (3) 9 (4) 6

Ans. (3)

Sol. Slope of the common tangent is m

Then Equation of tangent to Ellipse is $y = mx \pm \sqrt{16m^2+9}$

Equation of tangent to Circle is $y = mx \pm 2\sqrt{3}\sqrt{1+m^2}$

Since both tangents are identical

$\Rightarrow \quad 16m^2 + 9 = 12 + 12m^2$

$\Rightarrow \quad 4m^2 = 3$

$\Rightarrow \quad 12m^2 = 9$

7. If $\int\frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + C$, then the value of $g\left(\frac{1}{2}\right)$ is

(1) $\frac{\pi}{3}+\ln\left(2+\sqrt{3}\right)$ (2) $\frac{\pi}{6}+\ln\left(2+\sqrt{3}\right)$ (3) $\frac{\pi}{3}-\ln\left(2+\sqrt{3}\right)$ (4) $\frac{\pi}{6}-\ln\left(2+\sqrt{3}\right)$

Ans. (3)

Sol. $\int\frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx$ Put $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}\,dt$

$= \int\frac{-t}{t^2}\sqrt{\frac{t-1}{t+1}}dt$

$= -\int\frac{\sqrt{t-1}}{t\sqrt{t+1}}dt$

$= -\int\frac{t-1}{t\sqrt{t^2-1}}dt$

$= \int\frac{1}{t\sqrt{t^2-1}}dt - \int\frac{dt}{\sqrt{t^2-1}}$

$\sec^{-1}t - \ell n\left(t+\sqrt{t^2-1}\right)+C$

$\sec^{-1}\frac{1}{x} - \ell n\left(\frac{1}{x}+\sqrt{\frac{1}{x^2}-1}\right)+C$

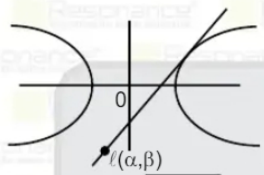
$g\left(\frac{1}{2}\right) = \sec^{-1}(2) - \ell n\left(2+\sqrt{2^2-1}\right)$

$= \frac{\pi}{3} - \ell n(2+\sqrt{3})$

8. The locus of point of intersection of any tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and a line perpendicular to the tangent and passing through (0,0) is $(x^2+y^2)^2 = \alpha x^2+\beta y^2$, then value of $(\alpha+\beta)$ is

Ans. **(12.00)**

Sol. $\frac{x^2}{16}-\frac{y^2}{4}=1$



tangent $y = mx \pm \sqrt{16m^2-4}$

Given $y=\frac{-x}{m}$

$\Rightarrow m=\frac{-x}{y}$

then $y = \frac{-x^2}{y}+\frac{\sqrt{16.x^2-4y^2}}{y}$

$y^2 + x^2 = \sqrt{16x^2-4y^2}$

$(y^2 + x^2)^2 = 16x^2 - 4y^2$

$\alpha = 16, \beta = -4$

the value of $\alpha + \beta = 12$

9. Area bounded between the curve $y^2 = 8x$ and $y^2 = 16(3-x)$ is :

(1) 32 (2) 42 (3) 48 (4) 16

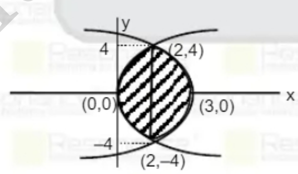
Ans. **(4)**

Sol. $y^2 = 8x$ & $y^2 = 16(3-x)$

Point of intersection $8x = 16(3-x)$

$x = 6-2x$

$\Rightarrow x = 2 \therefore y = \pm 4$



$A = 2\left(\int_0^2 2\sqrt{2}\sqrt{x}\,dx+\int_2^3 4\sqrt{3-x}\,dx\right)$

$$= 4\sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 + 8\left[-\frac{(3-x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^3$$

$$= \frac{8\sqrt{2}}{3}.2\sqrt{2} - \frac{16}{3}(0-1)$$

$$= \frac{32}{3} + \frac{16}{3} = \frac{48}{3} = 16$$

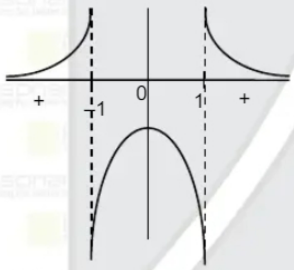
10. If $f(x) = x-1 : R \to R$ and $g(x) = \frac{x^2}{x^2-1} : R - \{-1,1\} \to R$ then $f(g(x))$ is

(1) one-one and onto function
(2) many-one and into function
(3) one-one and into function
(4) many-one and onto function

Ans. **(2)**

Sol. $f(g(x)) = f\left(\frac{x^2}{x^2-1}\right) \qquad x \neq \pm 1$

$= \frac{x^2}{x^2-1} - 1 = \frac{1}{x^2-1} \qquad x \neq \pm 1$



11. If ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20} = \frac{m}{n}\,({}^{60}C_{20})$ where m & n are co -prime then the value of m + n is

(1) 105
(2) 102
(3) 107
(4) 109

Ans. **(2)**

Sol. ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20}$

$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + ... + {}^{60}C_{40}$

$= {}^{41}C_{41} + {}^{41}C_{40} + {}^{42}C_{40} + ...$

$= {}^{42}C_{41} + {}^{42}C_{40} + ...$

$= {}^{60}C_{41} + {}^{60}C_{40} = {}^{61}C_{41} = \frac{61}{41} \times {}^{60}C_{40}$

$= \left(\frac{61}{41}\right)\,{}^{60}C_{20}$

So, m = 61, n = 41

m + n = 102

12. The equation of normal to the hyperbola $\frac{x^2}{\alpha} + \frac{x^2}{9} = 1$ at point $(8, 3\sqrt{3})$

(1) $4x + \sqrt{3}\,y = 41$
(2) $4x - \sqrt{3}\,y = 41$
(3) $\sqrt{3}\,x + 4y = 41$
(4) $\sqrt{3}\,x - 4y = 41$

Ans. (1)

Sol. $\because (8, 3\sqrt{3})$ lies on Hyperbola $\frac{x^2}{\alpha} + \frac{x^2}{9} = 1$

$\Rightarrow \frac{64}{\alpha} + \frac{27}{9} = 1 \Rightarrow \qquad \alpha = -32$

So equation of hyperbola is $-\frac{x^2}{32}+\frac{y^2}{9}=1$

Differentiating w.r.t. x $\Rightarrow -\frac{2x}{32}+\frac{2y}{9}\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{dy}{dx}=\frac{9x}{32y}$

Slope of normal at $(8,3\sqrt{3}) \Rightarrow -\frac{32(3\sqrt{3})}{9(8)} = -\frac{4\sqrt{3}}{3}$

Equation of normal $\Rightarrow y-3\sqrt{3} = -\frac{4}{\sqrt{3}}(x-8)$

$\Rightarrow \quad \sqrt{3}\,y-9=-4x+32$

$\Rightarrow \quad 4x+\sqrt{3}\,y=41$

13. The value of $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is-

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Ans. **(4)**

Sol. $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$

$\cos^{-1}\left(\frac{3}{10}\cos\left(\cos^{-1}\frac{3}{5}\right)+\frac{2}{5}\sin\left(\sin^{-1}\frac{4}{5}\right)\right)$

$\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}\right) = \cos^{-1}\left(\frac{9}{50}+\frac{8}{25}\right)$

$\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

14. There are 50 observation $x_1, x_2, x_3 \ldots x_{49}$, a. The mean and standard deviation of these observation are 15 and 2 respectively. Now if 'a' is replaced by b, then mean of these 50 observations so obtained is 16 then the variance of new 50 observations is (it is given than a + b = 70)

Ans. (46)

Sol. Given $\frac{x_1+x_2.........+x_{49}+a}{50}=15$

$= \sum x_i + a = 750$

$\therefore \sum x_i = 750 - a$

$\sigma^2 = \frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2$

$\Rightarrow 4=\frac{\sum x_i^2+a^2}{50}-225$

$\Rightarrow \sum x_i^2+a^2=11450$

New sum $\sum x_i + b = 750 - a + b$

$\therefore$ New mean $\frac{750-a+b}{50}=16$

$-a+b=800-750=50$

$a+b = 70$

$\therefore 2b = 120 \Rightarrow b=60\ \&\ a=10$

$\therefore$ New $\Sigma x^2 = 11450 - a^2 + b^2$

$= 14950$

$\therefore$ new $\sigma^2 = \frac{\sum x^2}{n}-(16)^2=\frac{14950}{50}-256$

$= 46$

15. If the compound statement $(r \vee \sim p) \rightarrow (p \wedge q) \vee r$ is tautology then r is equivalent to

(1) $\sim p$ (2) p (3) q (4) $\sim q$

Ans. (1)

Sol. $(r \vee \sim p) \rightarrow (p \wedge q) \vee r$

$\Rightarrow \sim(r \vee \sim p) \vee (p \wedge q) \vee r$

$\Rightarrow (\sim r \wedge p) \vee (p \wedge q) \vee r$

$\Rightarrow ((\sim r \wedge p) \vee r) \vee (p \wedge q)$

$\Rightarrow (\ldots (p \vee r)) \vee (p \wedge q)$

$\Rightarrow (p \wedge (p \vee r)) \vee (p \wedge q)$

$\Rightarrow (p \vee r) \vee (p \wedge q)$

$\Rightarrow r \vee (p \vee (p \wedge q))$

$\Rightarrow r \vee p$

will be a tautology if $r = \sim p$.

16. If the probability that 6 digits number formed using digits 8 and 1. Which is divisible by 21 is P then the value of 96 P is

Ans. (33)

Sol. No. of numbers formed by 8 and 1 of 6 digits which is divisible by 3 as well as 7 will contain three 8 and three 1 like given below .

8	8	8	1	1	1

1	8	1	8	1	8

and any number of 6 digits using same number will be divisible by 3 and 7 so two cases will arise like

1	1	1	1	1	1

&

8	8	8	8	8	8

$$\text{R. Prop} = P = \frac{\frac{\lfloor 6}{\lfloor 3 \lfloor 3} + 2}{2^6} = \frac{11}{32}$$

$$96\,P = 96 \times \frac{11}{32} = 33$$