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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 26 June, 2022 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHS

1. The function $f(x)=\frac{x-1}{x+1}$, $R-\{-1,0,-1\}$ and $f^{n+1}(x) = f(f^n(x))$, then the value of $f^6(6) + f^7(7)$ equals to

(1) $\frac{-3}{2}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

Ans. (01.00)

Sol. $f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = \frac{-2}{2x} = \frac{-1}{x}$

$f(f(f(f(x))) = f\left(f\left(-\frac{1}{x}\right)\right) = x$

$f^6(6) = \frac{1}{6}$, $f\left(-\frac{1}{x}\right) = f^7(x)$

$\frac{-\frac{1}{x}-1}{-\frac{1}{x}+1} = \frac{-1-x}{-1+x}$

$f^7(x) = \frac{x+1}{1-x}$

$f^7(7) = -\frac{4}{3}$

$f^6(6) + f^7(7) = -\frac{1}{6}-\frac{4}{3} = \frac{-1-8}{6} = \frac{-3}{2}$

2. If the line $\frac{x}{a}+\frac{y}{b}=2$ is tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at point (a,b), then find the value of n

(1) ϕ (2) singleton set (3) even (4) $n \in N$

Ans. **(04.00)**

Sol. Slope of tangent

$n\left(\frac{x}{a}\right)^{n-1}\left(\frac{1}{a}\right)+n\left(\frac{y}{b}\right)^{n-1}\left(\frac{1}{b}\right)\frac{dy}{dx}=0$

$\left(\frac{dy}{dx}\right)_{(a,b)} = -\frac{b}{a}$

equation of tangent $y-b = -\frac{b}{a}(x-a)$

$ay - ab = -bx + ab$

$bx + ay = 2ab$

$\Rightarrow$ for all $n \in N$

3. If F(x) = max (|x+1|,|x+2|,|x+3|,...|x+5|), then find the value of $\int_{-6}^{0} f(x)\,dx$.

Ans. **21.00**

Sol. $\int_{-6}^{-3}|x+1|\,dx+\int_{-3}^{0}|x+5|\,dx)$

$= -\int_{-6}^{-3}(x+1)dx+\int_{-3}^{0}(x+5)dx$

$= -\left(\frac{x^2}{2}+x\right)_{-6}^{-3}+\left(\frac{x^2}{2}+5x\right)_{-3}^{0}$

$= -\left(\frac{9}{2}-3-18+6\right)-\left(\frac{9}{2}-15\right)$

$= -\left(\frac{9-30}{2}\right)-\left(\frac{9-30}{2}\right)$

= 21

4. Evaluate $\lim_{x\to\frac{1}{\sqrt{2}}}\left(\frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}\right)$

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) –1 (4) 1

Ans. (1)

Sol. $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \left(\dfrac{\sin(\cos^{-1}x) - x}{1 - \tan(\cos^{-1}x)} \right)$

$= \lim\limits_{x \to \frac{1}{\sqrt{2}}} \left(\dfrac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right) - x}{1 - \tan\left(\tan^{-1}\dfrac{\sqrt{1-x^2}}{x}\right)} \right)$

$= \lim\limits_{x \to \frac{1}{\sqrt{2}}} \left(\dfrac{\sqrt{1-x^2} - x}{1 - \dfrac{\sqrt{1-x^2}}{x}} \right)$

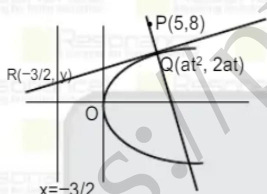
$= \operatorname{limit}\limits_{x \to \frac{1}{\sqrt{2}}} (-x) = -\dfrac{1}{\sqrt{2}}$

5. A normal is drawn from P(5, 8) to the parabola $y^2 = 6x$ meeting it at Q. Tangent at Q to $y^2 = 6x$ meet at point R on the directrix, then ordinate of point R is

(1) $\frac{3}{2}$ (2) $\frac{9}{2}$ (3) $\frac{9}{4}$ (4) $\frac{6}{2}$

Ans. (3)

Sol.



Equation of normal to parabola

$y = -tx + 2at + at^3$

$y = -tx + 3t + \frac{3}{2}t^3 \quad \because a = \frac{3}{2}$

$8 = -5t + 3t + \frac{3}{2}t^3$

$\Rightarrow \frac{3}{2}t^3 - 2t - 8 = 0$

$\Rightarrow 3t^3 - 4t - 16 = 0$

$\Rightarrow (t-2)(3t^2 + 6t + 8) = 0$

So, $t = 2$

As, $t = 2$, we get Q(6,6)

Equation of tangent to parabola $y^2 = 6x$

$yt = x + at^2$

$2y = x + \frac{3}{2} \times 4$

$2y = x + 6$ ....(1)

put $x = \frac{-3}{2}$ in equation (1)

$\Rightarrow 2y = \frac{-3}{2} + 6 = \frac{9}{2}$

$y = \frac{9}{4}$

6. If A is a matrix of order 3 x 3 and if |adj (24A)| = |adj (3adj 2A)|, then $|A|^2$ is equal to :

(2) 2^6 (3) 2^{12} (4) 6^6

Ans. (2)

Sol. $|24A|^2 = |3adj2A|^2$

$\Rightarrow 24^6 |A^2| = 3^6 |2A|^4$

$\Rightarrow 8^6 |A|^2 = 2^{12} |A|^4$

$\Rightarrow |A|^2 = 2^6$

7. Let common tangent to $x^2 + y^2 = \frac{9}{4}$ and $y^2 = 4x$ meet x-axis at point Q. If an ellipse with length of major axis equal to 6 and minor axis is of length 'OQ' is drawn, then the value of $\frac{\ell}{e^2}$ is

(Where ℓ is length of latus rectum and 'e' is eccentricity of the ellipse and O is origin)

(1) 2 (2) 5 (3) 7 (4) 8

Ans. (1)

Sol. $y = mx \pm \frac{3}{2}\sqrt{1+m^2}$ ......(i)

$y = mx + \frac{1}{m}$ ......(ii)

comparing (i) and (ii)

$\frac{3}{2}\sqrt{1+m^2} = \frac{1}{m}$

$\Rightarrow 9(m^2 + m^4) = 4$

Put, $m^2 = t$

$9t^2 + 9t - 4 = 0$

$(3t)^2 + 3.(3t) - 4 = 0$

$(3t + 4)(3t - 1) = 0$

$m^2 = -\frac{4}{3}, \frac{1}{3}$

Let $m = \frac{1}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ if $y = 0 \Rightarrow x = -3 \Rightarrow OQ = 3$

Now $2a = 6$ and $2b = 3$ $\quad b^2 = a^2(1 - e^2)$

$\frac{\ell}{e^2} = \frac{\frac{2b^2}{a}}{1-\frac{b^2}{a^2}} = \frac{2ab^2}{a^2 - b^2}$

$= \frac{6\times\frac{9}{4}}{9-\frac{9}{4}} = \frac{\frac{6}{4}}{1-\frac{1}{4}} = \frac{6}{3} = 2$

8. In how many ways a team of 3 boys and 3 girls can be formed from 10 boys and 5 girls, if two particular boys B_1 and B_2 are not in team together :

(1) 1115 (2) 1120 (3) 1280 (4) 1220

Ans. **(2)**

Sol. Total no. of way to select 3 boys and 3girls = $^{10}C_3 . ^5C_3 = 1200$

When particular boys B_1 and B_2 both are in team = $^8C_1 . ^5C_3 = 80$

So required number of ways = 1200 – 80 = 1120

9. If $(2021)^{2023}$ is divided by 7, then remainder is equal to

Ans. **(05.00)**

Sol. $(288 \times 7 + 5)^{2023}$

$= 7\lambda + 5^{2023}$

$= 7\lambda + (7 - 2)^{2023}$

$= 7\lambda + 7\mu - 2^{2023}$

$= 7\lambda + 7\mu - 2.\ 8^{674}$

$= 7\lambda + 7\mu - 2\ (7 + 1)^{674}$

$= 7\lambda + 7\mu - 2(7I + 1)$

$= 7m - 2 = 7(m - 1) + 7 - 2 = 7\ (m - 1) + 5$

where λ, μ, I, m are integers

10. A biased coin is tossed 5 times and probability of getting exactly 4 heads is equal to probability of getting exactly 5 heads, then the probability of getting at most 2 heads, is

(1) $\frac{27}{648}$ (2) $\frac{25}{648}$ (3) $\frac{23}{648}$ (4) $\frac{24}{648}$

Ans. **(3)**

Sol. $^5C_4 . p^4.q = {^5C_5} . p^5$

5q= p, & p + q = 1

So 5q = 1–q

$\Rightarrow q = \frac{1}{6}, \quad p = \frac{5}{6}$

P(at most two heads) = P(x=0) + P(x=1) + P(x=2)

$^5C_0 .q^5 + 5C_1. p. q^4 + {^5C_2} . p^2 . q^3$

$q^5 + 5.pq^4 + 10.\ p^2 .q^3$

$= q^3 (q^2 + 5qp + 10p^2)$

$= \frac{1}{6^3}\left(\frac{1}{6^2} + 5.\frac{5}{6^2} + 10.\frac{25}{6^2}\right)$

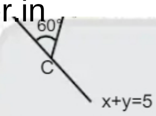
$= \frac{276}{6^5} = \frac{46}{6^4} = \frac{23}{3.6^3} = \frac{23}{648}$

11. Triangle ABC is an equilateral triangle with vertex A(3, 7) & B, C lie on line x + y = 5. Area of ΔABC is

(1) $\frac{25}{2\sqrt{3}}$ (2) $\frac{25}{\sqrt{6}}$ (3) 25 (4) $\frac{25}{\sqrt{2}}$

Ans. **(1)**

Sol.



$$h = \frac{|3+7-5|}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$\ell \sin 60^\circ = h$$

$$\ell = \frac{2h}{\sqrt{3}}$$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}\ell^2 = \frac{\sqrt{3}}{4} \times \frac{4h^2}{3}$

$$= \frac{1}{\sqrt{3}} \times \left(\frac{5}{\sqrt{2}}\right)^2 = \frac{25}{2\sqrt{3}}$$

12. If $A = \sum_{j=1}^{10}\sum_{i=1}^{10} \min(i,j)$, $B = \sum_{i=1}^{10}\sum_{j=1}^{10} \max(i,j)$, then A + B is equal to

(1) 1000 (2) 1100 (3) 1200 (4) 1190

Ans. **(2)**

Sol. (1,1) , (1,2) ...., (1,10)
(2,1) , (2,2) ...., (2,10)
(3,1) , (3,2) ...., (3,10)
⋮
(10,1) , (10,2) ...., (10,10)

$A + B = 2.(9 \times 1 + 8 \times 2 + 7 \times 3 + \ldots\ldots + 2 \times 8 + 1 \times 9)$
$+ 2\,(2 \times 1 + 3 \times 2 + 4 \times 3 + \ldots\ldots + 9 \times 8 + 10 \times 9)$

$+ 2\,(1 + 2 + 3 + \ldots + 10)$

$= 2.(11 \times 9 + 11 \times 8 + 11 \times 7 + \ldots\ldots + 11 \times 1) + 2 \times \frac{10 \times 11}{2}$

$= 2 \times 11 \times \frac{9 \times 10}{2} + 10 \times 11 = 10 \times 11 \times 10 = 1100$

13. If five observations a,b,5,8,10 has mean and variance as 6 and 6.8 respectively, then the value of (mean deviation about mean) × 25 is ....

Ans. **(60.00)**

Sol. $a + b + 23 = 30 \Rightarrow a + b = 7$ ....(1)

$6.8 = \frac{1}{5} \times \left(a^2 + b^2 + 25 + 64 + 100\right) - 36$

$34 = a^2 + b^2 + 189 - 180$

$a^2 + b^2 = 25$ ....(2)

from equation (1) & (2)

a = 4, b = 3 or a = 3, b = 4

$M.D. = \frac{\Sigma |x_i - 6|}{5} = \frac{12}{5}$

So answer is 60

14. Sum of the cube of the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is equal to :

Ans. **(36)**

Sol. $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$

$\therefore x \neq 0$ so divided by x^2

$\Rightarrow x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$

$\Rightarrow x^2 + \frac{1}{x^2} - 3\left(x - \frac{1}{x}\right) - 2 = 0$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right) = 0$

$\Rightarrow x-\frac{1}{x}=3$ or $x-\frac{1}{x}=0$

$\Rightarrow x^2-3x-1=0$ (Let roots of $x^2-3x-1=0$ are α & β)

and $x^2-1=0$ (Let roots of $x^2-1=0$ are γ & δ)

$\alpha+\beta=3$ $\qquad$ $x=\pm 1$

$\alpha\beta=-1$ $\qquad$ Let $\gamma=1$, $\delta=-1$

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=36$

So required sum $=\alpha^3+\beta^3+\gamma^3+\delta^3=36+1-1=36$

15. If $\vec a\cdot\vec b=1$, $\vec b\cdot\vec c=2$, $\vec c\cdot\vec a=3$ then the value of $\left[\vec a\times(\vec b\times\vec c),\ \vec b\times(\vec a\times\vec c),\ \vec c\times(\vec a\times\vec b)\right]$ is

Ans. **(00.00)**

Sol. $\left[(\vec a\cdot\vec c)\vec b-(\vec a\cdot\vec b)\vec c \quad (\vec b\cdot\vec c)\vec a-(\vec b\cdot\vec a)\vec c \quad (\vec c\cdot\vec b)\vec a-(\vec c\cdot\vec a)\vec b\right]$

$=\left[3\vec b-\vec c \quad 2\vec a-\vec c \quad 2\vec a-3\vec b\right]$

$=\begin{vmatrix}0 & 3 & -1\\ 2 & 0 & -1\\ 2 & -3 & 0\end{vmatrix}\left[\vec a\ \vec b\ \vec c\right]$

$=6\begin{vmatrix}0 & 1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{vmatrix}\left[\vec a\ \vec b\ \vec c\right]=6.(0).\left[\vec a\ \vec b\ \vec c\right]=0$

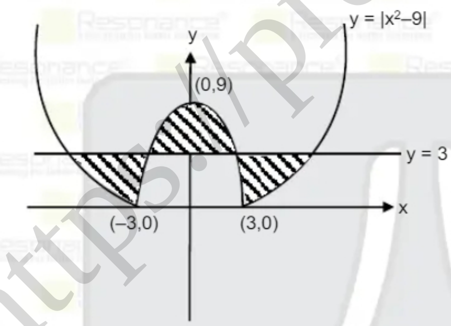
16. Area bounded between the curves $y=|x^2-9|$ and $y=3$ is equal to :

(1) $8(4\sqrt3-2\sqrt6-9)$ (2) $8(2\sqrt6-4\sqrt3-9)$

(3) $8(4\sqrt3+2\sqrt6-9)$ (4) $4(4\sqrt3+2\sqrt6-9)$

Ans. **(3)**

Sol.



Area$=2\left(\int_0^3\left(\sqrt{y+9}-\sqrt{9-y}\right)dy+\int_3^9\sqrt{9-y}\,dy\right)$

$=2\left(\int_0^3\sqrt{y+9}\,dy+\int_0^9\sqrt{9-y}\,dy-2\int_0^3\sqrt{9-y}\,dy\right)=\frac{4}{3}\left[(y+9)^{\frac32}+2(9-y)^{\frac32}\right]_0^3-\frac{4}{3}\left[(9-y)^{\frac32}\right]_0^9$

$=\frac43\left[12^{\frac32}+2.6^{\frac32}-2.9^{\frac32}\right]=\frac43\left[12\times2\sqrt3+2.6\sqrt6-2.27\right]=\frac43\left[24\sqrt3+12\sqrt6-54\right]$

$=4\left[8\sqrt3+4\sqrt6-18\right]=8\left[4\sqrt3+2\sqrt6-9\right]$

17. If $f(x)=2\cos^{-1}x+4\cot^{-1}x-3x^2-2x+10$, the range of f(x) is [a,b] , then the value of 4a – b is

(1) $11-\pi$ (2) $11+\pi$ (3) 11 (4) π

Ans. **(1)**

Sol. $f(x)=2\cos^{-1}x+4\cot^{-1}x-3x^2-2x+10$

$f'(x)<0$

so the function f(x) is decreasing

the domain of function is [–1,1]

$f(-1)=2\pi+3\pi-3+2+10$

$=9+5\pi=b$

$f(1) = \pi - 3 - 2 + 10 = 5 + \pi = a$

$4a - b = 4\pi + 20 - 9 - 5\pi = 11 - \pi$