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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 24 June, 2022 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. The value of ... dx is equal to :

$\int_{-\pi/2} \frac{dx}{(1+e^x)(\sin^6 x+\cos^6 x)}$ is equal to :

(1) 0 (2) 2π (3) π (4) $\frac{\pi}{2}$

Ans. (3)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{(1+e^x)(\sin^6 x+\cos^6 x)}$

$= \int_0^{\pi/2} \left(\frac{1}{(1+e^x)(\sin^6 x+\cos^6 x)} + \frac{1}{(1+e^{-x})(\sin^6 x+\cos^6 x)} \right) dx$

$= \int_0^{\pi/2} \frac{dx}{\sin^6 x+\cos^6 x}$

$= \int_0^{\pi/2} \frac{dx}{\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x}$

$= \int_0^{\pi/2} \frac{\sec^4 x}{\tan^4 x-\tan^2 x+1} dx$

$= \int_0^{\pi/2} \frac{(1+\tan^2 x)(\sec^2 x)}{\tan^4 x-\tan^2 x+1} dx$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$= \int_0^{\infty} \frac{(1+t^2)dt}{t^4-t^2+1} = \int_0^{\infty} \frac{(1+\frac{1}{t^2})}{t^2-1+\frac{1}{t^2}} dt$

$= \int_0^{\infty} \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+1} dt$

$= \left[\tan^{-1}\left(t-\frac{1}{t}\right)\right]_0^{\infty}$

$= \frac{\pi}{2}+\frac{\pi}{2}$

$= \pi$

2. If $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right) = \frac{1}{4}\cos^2(2x)$ then the number of solutions in the interval $[-3\pi, 3\pi]$ is

Sol. (07.00)

Sol. $\cos^2 x - \sin^2 \frac{\pi}{3} = \frac{1}{4}\cos^2 2x$

$\frac{1+\cos 2x}{2} - \frac{3}{4} = \frac{1}{4}\cos^2 2x$

Now put the $\cos 2x = t$

$-1+2t = t^2 \quad \Rightarrow t = 1$

$\cos 2x = 1 \quad \Rightarrow x = n\pi$

hence 7 solutions in the interval $[-3\pi, 3\pi]$

3. A tangent of a curve at point P intersect x axis at point Q and y-axis bisect line segment PQ. If curve passes through the point (3,3) then equation of curve is

(1) $xy = 3$ (2) $y^2 = 3x$ (3) $y^2 = 5x$ (4) $xy = 5$

Ans. **(2)**

Sol. Tangent at P (x,y) is

$Y - y = m(X - x)$

point at x -axis is $Q\left(x - \frac{y}{m}, 0\right)$

point at y axis is R $(0, y - mx)$

Now R is mid point at PQ

$\therefore\ O = \frac{x + x - \frac{y}{m}}{2}$

$\Rightarrow 2x = \frac{y}{m} \Rightarrow \frac{dy}{dx} = \frac{y}{2x}$

$\Rightarrow \frac{2}{y}dy = \frac{1}{x}dx$

$\Rightarrow 2\ln y = \ln x + \ln c$

$\Rightarrow y^2 = cx$

it passes through (3,3)

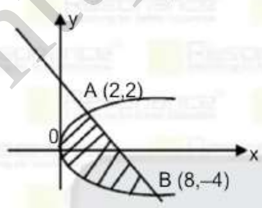
$\therefore\ 9 = 3c \Rightarrow c = 3$

$\therefore\ y^2 = 3x$

4. Area enclosed between the curves $y^2 = 2x$ and $x + y = 4$ is

Ans. **18.00**

Sol. On solving both equations



$\frac{y^2}{2} + y = 4$

$y^2 + 2y - 8 = 0 \Rightarrow y = -4, y = 2$

so A(2, 2) ; B(8–4)

Required area $= \int_{-4}^{2}\left((4 - y) - \frac{y^2}{2}\right)dy$

$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2} = 18$ sq. unit

5. The value of $\lim \left(\frac{n^2}{(\ldots)} + \frac{n^2}{(\ldots)} + \ldots + \frac{n^2}{(\ldots)}\right)$ is

(1) $\frac{2\ell n2+\pi}{8}$ (2) $\frac{3\ell n2+\pi}{8}$ (3) $\frac{4\ell n2+\pi}{8}$ (4) $\frac{5\ell n2+\pi}{8}$

Ans. (1)

Sol. Given $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\left(1+\frac{r^2}{n^2}\right)\left(1+\frac{r}{n}\right)}$$

$$\int_0^1\frac{dx}{(1+x^2)(1+x)}$$

$$=\frac{1}{2}\int_0^1\frac{dx}{x+1}+\frac{1}{2}\int_0^1\frac{1-x}{x^2+1}dx$$

$$\left[\frac{1}{2}\ell n(x+1)+\frac{1}{2}\tan^{-1}x-\frac{1}{4}\ell n(x^2+1)\right]_0^1$$

$$\frac{1}{2}\left(\ell n2+\frac{\pi}{4}-\frac{1}{2}\ell n2\right)$$

$$=\frac{1}{2}\left(\frac{1}{2}\ell n2+\frac{\pi}{4}\right)=\frac{1}{4}\ell n2+\frac{\pi}{8}$$

6. Let $f_\lambda(x) = 4\lambda x^3 - 8\lambda x^2 + 36x + 48$ be a real function. It λ^* is the greatest integral value of λ for which $f_\lambda(x)$ is increasing function then the value $f_{\lambda^*}(1)+f_{\lambda^*}(-1)$ is equal to

(1) 0 (2) 1 (3) –1 (4) 2

Ans. (1)

Sol. $\because f_\lambda'(x)\ge 0, \forall x\in R$

$\Rightarrow 12\lambda x^2 - 16\lambda x + 36 \ge 0\ x\in R$

$\Rightarrow 3\lambda x^2 - 4\lambda x + 9 \ge 0, \forall x\in R$

$\Rightarrow$ either $\lambda = 0$ or $3\lambda > 0$ and $4^2\lambda^2 - 4.3\lambda.9 \le 0$

$\Rightarrow 0 \le \lambda \le \frac{27}{4}$

$\Rightarrow \lambda^* = 6$

$\Rightarrow f_6(1) + f_6(-1) = (24-48+36+48) + (-24-48-36+48) = 0$

7. If $(1 + 3 + 3^2 + \ldots\ldots + 3^{2021})$ is divided by 50, then remainder is

Ans. (04.00)

Sol. $1 + 3 + 3^2 + \ldots\ldots 3^{2021} = \frac{3^{2022}-1}{2} = \frac{9^{1011}-1}{2}$

$\Rightarrow \frac{(10-1)^{1011}-1}{2} = \frac{100\lambda + {}^{1011}C_{1010}\times 10 - 2}{2} = 50\lambda + 5054$

when divided by 50 gives remainder = 4

8. Number of real roots of equation $x^7 - 7x - 2 = 0$ is/are

(1) 1 (2) 3 (3) 5 (4) 7

Sol. $f(x) = x^7 - 7x - 2$

$f'(x) = 7x^6 - 7$

$= 7(x^6 - 1)$

$= 7(x^3 + 1)(x^3 - 1)$

$f'(x) = 7(x+1)(x-1)(x^2+x+1)(x^2-x+1)$

$f'(x) = 0$ has two real roots

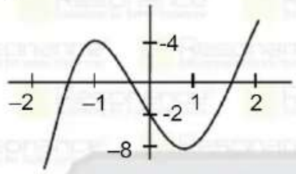
$f(1) = -8$

$f(-1) = 4$

$f(2) = 112$

$f(-2) = -116$

$f(0) = -2$



So by graph $f(x) = 0$ has three real roots

9. If HCF (a, 24) =1 and a $\in$ {1,2,3,.......100}, then the sum of all values of a is

(1) 1634 (2) 1633 (3) 1635 (4) 1636

Ans. **(2)**

Sol. Let n(a) = number of numbers divisible by a , so

$n(2) = \{2,4,6......100\} \Rightarrow 50$ numbers

$n(3) = \{3,6,9......99\} \Rightarrow 33$ numbers

$n(2 \cap 3) = \{6,12,18......96\} \Rightarrow 16$ numbers

$\Rightarrow$ Sum of all numbers divisible by 2 = $\frac{50}{2}[2+100] = 50 \times 51$

Sum of all numbers divisible by 3 = $\frac{33}{2}[3+99] = 33 \times 51$

Sum of all numbers divisible by 6 = $\frac{16}{2}[6+96] = 16 \times 51$

Sum of all numbers divisible by either 2 or 3 = $50 \times 51 + 33 \times 51 - 16 \times 51 = 67 \times 51$

Sum of all natural numbers form 1 to 100 = $\frac{100}{2}[1+100] = 50 \times 101$

Sum of required values of 'a' is $101 \times 50 - 67 \times 51 = 1633$

10. If x and y are two positive numbers such that $x^3y^2 = 2^{15}$ then the least value of 3x+ 2y is

(1) 30 (2) 40 (3) 50 (4) 45

Ans. **(2)**

Sol. We know that A.M $\geq$ G.M.

$\therefore \frac{x+x+x+y+y}{5} \geq \sqrt[5]{x^3.y^2}$

$\Rightarrow 3x + 2y \geq 5.(2^{15})^{\frac{1}{5}}$

$\Rightarrow 3x + 2y \geq 40$

11. Let A : Rajnish is a Judge

B : Rajnish is Honest

C : Rajnish is not Arrogant

Then negation of "If Rajnish is a Judge and not Arrogant then he is Honest" is

(1) $\sim(A \wedge C) \wedge B$ (2) $\sim B \wedge (A \wedge C)$ (3) $\sim B \vee (A \wedge C)$ (4) $\sim B \wedge (A \vee C)$

Ans. (2)

Sol. Given statement is $(A \wedge C) \rightarrow B$

$\therefore$ Negation is $\sim((A \wedge C) \rightarrow B)$

$(A \wedge C) \wedge \sim B = \sim B \wedge (A \wedge C)$

12. If the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{1} = 1$ and the ellipse $3x^2 + 4y^2 = 12$ have same latusrectum, then the value of $12\left(e_H^2 + e_E^2\right)$ is equal to (where e_H and e_E are eccentricities of hyperbola and ellipse respectively)

Sol. given $\frac{2(1)}{a} = \frac{2(3)}{2}$

$\Rightarrow a = 2/3$

So $e_H = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

$e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

So, $12\left(e_H^2 + e_E^2\right) = 12\left(\frac{13}{4} + \frac{1}{4}\right) = 42$

13. If $y = \tan^{-1}(\sec x^3 - \tan x^3)$ and $\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$ then which of the following is correct

(1) $xy'' + 2y' = 0$ (2) $x^2y'' - 6y + \frac{3\pi}{2} = 0$ (3) $x^2y'' - 6y + 3\pi = 0$ (4) $xy'' - 4y' = 0$

Ans. (2)

Sol. $y = \tan^{-1}\left(\frac{1 - \sin x^3}{\cos x^3}\right)$

$= \tan^{-1}\left[\frac{1 - \cos\left(\frac{\pi}{2} - x^3\right)}{\sin\left(\frac{\pi}{2} - x^3\right)}\right]$

$= \tan^{-1}\left[\frac{2\sin^2\left(\frac{\pi}{4} - \frac{x^3}{2}\right)}{2\sin\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x^3}{2}\right)}\right] = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\right)$

$y = \frac{\pi}{4} - \frac{x^3}{2}$ $\left(\because -\frac{\pi}{2} < \frac{\pi}{4} - \frac{x^3}{2} < 0\right)$

$= y' = -\frac{3}{2}x^2$

$= y'' = -3x$

option (1) $xy'' + 2y' = -3x^2 + 2\left(\frac{-3}{2}x^2\right) = -6x^2$ hence 1 is incorrect

option (2) $x^2y'' - 6y + \frac{3\pi}{2} = x^2(-3x) - 6\left(\frac{\pi}{4} - \frac{x^3}{2}\right) + \frac{3\pi}{2} = 0$ option 2 is correct

similarly check other option

14 If a complex number $z = \alpha + i\beta$ satisfy $|z-3|^2 \le 1$ and $z(4+3i) + \bar{z}(4-3i) \le 24$ and its distance from point $-4i$ is minimum then the value of $25(\alpha+\beta)$ is

Ans. **(40)**

Sol. $|z-3|^2 \le 1$

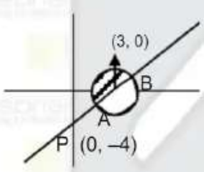
$0 \le |z-3|^2 \le 1$

$(\alpha-3)^2 + \beta^2 \le 1$ ......(1)

$Z(4+3i) + \bar{z}(4-3i) \le 24$

$2(4\alpha - 3\beta) \le 24$

$4\alpha - 3\beta \le 12$ ......(2)



Point of intersection of (1) & (2)

$(\alpha-3)^2 + \left(\frac{4\alpha-12}{3}\right)^2 = 1$

$25\alpha^2 - 150\alpha + 216 = 0$

$\alpha = \frac{12}{5}, \frac{18}{5}$ ($\because$ distance from point $-4i$ is minimum so we take $\alpha = \frac{12}{5}$)

Hence $P\left(\frac{12}{5}, -\frac{4}{5}\right) = (\alpha, \beta)$

Now $25(\alpha+\beta) = 25\left(\frac{12}{5} - \frac{4}{5}\right) = 40$

15. If probability distribution of random experiment 'x' is as following

x	0	1	2	3	4
P(x)	k	2k	3k	4k	5k

then the value of $P((1 < x < 4) | x \le 2)$ is

Sol. $P(1 < x < 4 \mid x \le 2) = \frac{3k}{6k} = \frac{1}{2}$

16. The number of 7 digit numbers which are divisible by 11 formed by 1,2,3,5,6,7 & 9 is

Ans. **(288)**

Sol. Let number is $a_1a_2a_3a_4a_5a_6a_7$

By divisibility test of 11 we can say

$(a_2 + a_4 + a_6) - (a_1 + a_3 + a_5 + a_7) = 0$ or 11 or -11

(i) $a_1 + a_3 + a_5 + a_7 = a_2 + a_4 + a_6$

Not possible as sum of given digit is 33

(ii) $a_1 + a_3 + a_5 + a_7 - (a_2 + a_4 + a_6) = 11$

$\Rightarrow a_1 + a_3 + a_5 + a_7 = 11 + a_2 + a_4 + a_6$

∴	a_2	a_4	a_6	
	1	3	7	(3! ways)
or	2	3	6	(3! ways)

∴ Total ways in this case = $2\lfloor 3 \lfloor 4 = 288$

(iii) $a_1 + a_3 + a_5 + a_7 - (a_2 + a_4 + a_6) = -11$

$\Rightarrow a_1 + a_3 + a_5 + a_7 + 11 = a_2 + a_4 + a_6$

∴	a_2	a_4	a_6	
	6	7	9	(3! ways)

∴ Total ways in this case = $\lfloor 3 \lfloor 4 = 144$

total case 288 + 144 = 432

17. If $x * y = x^2 - y^3$ such that $(x * 1) * 1 = 1 * (x * 1)$, then the value of $2\sin^{-1}\left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2}\right)$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. **(3)**

Sol. $(x * 1) * 1 = 1 * (x * 1)$

$(x^2 - 1)^2 - 1 = 1 - (x^2 - 1)^3$

Put $x^2 - 1 = t$

$\Rightarrow t^3 + t^2 - 2 = 0$

$\Rightarrow (t - 1)(t^2 + 2t + 2) = 0$

$\Rightarrow t = 1$

$\Rightarrow x^2 - 1 = 1$

$\Rightarrow x^2 = 2$

So that $2\sin^{-1}\left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2}\right) = 2\sin^{-1}\left(\frac{4 + 2 - 2}{4 + 2 + 2}\right)$

$= 2\sin^{-1}\left(\frac{1}{2}\right)$

$= 2 \cdot \frac{\pi}{6}$

$= \frac{\pi}{3}$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