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JEE (Main) PAPER-1 (B.E./B. TECH.)

2022


COMPUTER BASED TEST (CBT) Memory Based Questions & Solutions

Date: 24 June, 2022 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)
Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1 If α and β are roots of the equation $3x^2 + \lambda x - 1 = 0$ such that $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$ then value of $6(\alpha^3 + \beta^3)^2$ is

- (1) 22 (2) 24 (3) 27 (4) 30

Ans. (2)

$$\alpha + \beta = \frac{-\lambda}{3} \quad \alpha\beta = -\frac{1}{3}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15 \Rightarrow \alpha^2 + \beta^2 = 15\alpha^2\beta^2$$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 15(\alpha\beta)^2$$

$$\Rightarrow \frac{\lambda^2}{9} - 2\left(-\frac{1}{3}\right) = 15\left(-\frac{1}{3}\right)^2$$

$$\Rightarrow \frac{\lambda^2}{9} + \frac{2}{3} = \frac{5}{3} \Rightarrow \lambda^2 = 9$$

$$6(\alpha^3 + \beta^3)^2 = 6((\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta))^2$$

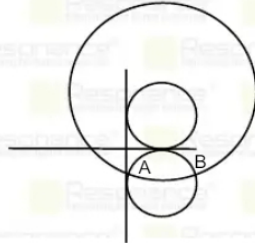
$$= 6\left(-\frac{\lambda^3}{27} + \left(-\frac{\lambda}{3}\right)\right)^2$$

$$= 6\frac{\lambda^2}{9}\left(\frac{\lambda^2}{9} + 1\right)^2$$

$$= 6 \times \frac{9}{9} \times \left(\frac{9}{9} + 1\right)^2 = 6 \times 4 = 24$$

2. If $S = \{z \in \mathbb{C} : 1 \leq |z - (1 + i)| \leq 2\}$ and $A = \{z \in S : |z - (1 - i)| = 1\}$ then A is
 (1) null set
 (2) singleton set
 (3) exactly two element
 (4) infinite elements

Ans. (4)
 Sol.



Set A contains all points on minor arc AB of circle $|z - (1 - i)| = 1$

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3. Let S be the solution set of $\sin\theta \tan\theta + \tan\theta = \sin^2\theta$.
 Such that $\theta \in (-\pi, \pi)$, $\theta \neq \pm \frac{\pi}{2}$ and $t = \sum \cos 2\theta$, where θ is solution of above equation then value of

$t + n(S)$ is.
 (1) 8 (2) 5 (3) 6 (4) 9

Ans. (2)

Sol. $\sin\theta \tan\theta + \tan\theta = \sin^2\theta$
 $\tan\theta (\sin\theta + 1) - 2 \sin\theta \cos\theta = 0$
 $\sin\theta \left(\frac{\sin\theta + 1}{\cos\theta} - 2 \cos\theta \right) = 0$

$$\sin\theta = 0 \text{ or } \sin\theta + 1 - 2 \cos^2\theta = 0$$

$$\theta = 0 \quad \sin\theta + 1 - 2(1 - \sin^2\theta) = 0$$

$$2 \sin^2\theta + \sin\theta - 1 = 0$$

$$\sin\theta = -1, \frac{1}{2}$$

$$\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

Hence, $S = \{0, \frac{\pi}{6}, \frac{5\pi}{6}\} \Rightarrow n(S) = 3$

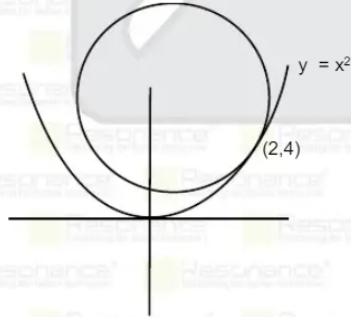
$$t = \sum (\cos(2\theta)) = \cos(0) + \cos(\pi/3) + \cos(5\pi/3)$$

$$= 1 + \frac{1}{2} + \frac{1}{2} = 2$$

4. A circle of equation $x^2 + y^2 + Ax + By + C = 0$ passes through (0,6) and touches the parabola $y = x^2$ at (2,4) then A + C is

(1) 16 (2) 25 (3) 32 (4) 4

Ans. (1)
Sol.



Equation of tangent to parabola at (2,4) $y - 4 = m(x - 2)$

$$\frac{dy}{dx} = 2x \Rightarrow m = \frac{dy}{dx}\bigg|_{(2,4)} = 4$$

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$$y - 4 = 4(x - 2) \Rightarrow 4x - y - 4 = 0$$

Let equation of circle be $S + \lambda L = 0$

$$(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$$

It passes through (0,6) $\Rightarrow 4 + 4 + \lambda(0 - 6 - 4) = 0$

$$\lambda = \frac{8}{10} = \frac{4}{5}$$

$$(x - 2)^2 + (y - 4)^2 + \frac{4}{5}(4x - y - 4) = 0$$

$$x^2 + y^2 + \left(\frac{16}{5}x - 4x\right) + \left(-8y - \frac{4}{5}y\right) + \left(20 - \frac{16}{5}\right) = 0$$

$$x^2 + y^2 - \frac{4}{5}x - \frac{44y}{5} + \frac{84}{5} = 0$$

$$A = -\frac{4}{5}, B = -\frac{44}{5}, C = \frac{84}{5}$$

$$A + C = \frac{80}{5} = 16$$

5. If $x(y) = x$ and $y \frac{dx}{dy} = 2x + y^3(y + 1)e^y$ and $x(1) = 0$, then $x(e)$ is equal to

(1) $(e^e - 1)$ (2) $e^3(e^e - 1)$ (3) $e^e - 1$ (4) $e^e - 3$

Ans. (2)

Sol. $\frac{dx}{dy} - \frac{2x}{y} = y^2(y + 1)e^y$

$$\text{I.F.} = e^{\int -\frac{2}{y} dy} = e^{-2 \ln y} = \frac{1}{y^2}$$

$$\frac{x}{y^2} = \int (y + 1)e^y dy$$

$$\frac{x}{y^2} = y \cdot e^y + C$$

$$0 = e + C \Rightarrow C = -e$$

$$\Rightarrow \frac{x}{y^2} = ye^y - e \Rightarrow \frac{x}{e^2} = e \cdot e^e - e$$

$$\Rightarrow x = e^3(e^e - 1)$$

6. A balloon, spherical in shape is inflated and its surface area is increasing with a constant rate, initially the radius is 3 units, after 5 sec radius is 7 units then the radius after 9 second is

(1) 9 (2) 7 (3) 5 (4) 3

Sol. $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = k$$

$$4\pi r^2 = kt + C$$

$$\text{at } t = 0, r = 3 \Rightarrow 36\pi = C$$

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$$\text{at } t = 5, r = 7 \Rightarrow 4\pi \times 49 = 5k + 36\pi$$

$$\Rightarrow 5k = 4\pi(49 - 9)$$

$$5k = 4\pi \times 40$$

$$k = 32\pi$$

$$\Rightarrow 4\pi r^2 = 32\pi t + 36\pi$$

$$\Rightarrow r^2 = 8t + 9 \Rightarrow r^2 = 81 \Rightarrow r = 9$$

7. If \hat{a} & \hat{b} are unit vectors and vectors \hat{c} & \hat{a} have an angle of $\frac{\pi}{12}$ between them such that $\hat{b} = \hat{c} + 2(\hat{c} \times \hat{a})$

then the value of $|\hat{b}|^2$ is.

(1) $3 + \sqrt{3}$

(2) $3 - \sqrt{3}$

(3) $2 + \sqrt{3}$

(4) $2 - \sqrt{3}$

Ans. (1)

Sol. $|\hat{b}| = \sqrt{c^2 + 4(\hat{c} \times \hat{a})^2 + 0}$

$$1 = c^2 + 4c^2 \sin^2 \frac{\pi}{12}$$

$$1 = c^2 + 4c^2 \times \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$$

$$1 = c^2 + c^2 \left(\frac{4-2\sqrt{3}}{2}\right)$$

$$1 = c^2 + c^2(2-\sqrt{3})$$

$$1 = c^2(3-\sqrt{3})$$

$$3+\sqrt{3} = c^2(3-\sqrt{3})(3+\sqrt{3})$$

$$3+\sqrt{3} = 6c^2$$

8. If $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ $x \in [0, 1]$, then the sum of absolute maxima and absolute minima is.

(1) $3 + \frac{1}{2}(\sin 2 + \sin 1)$

(2) $3 - \frac{1}{2}(\sin 2 + \sin 1)$

(3) $3 - \frac{1}{2}(\sin 2 - \sin 1)$

(4) $3 + \frac{1}{2}(\sin 2 - \sin 1)$

Ans. (1)

Sol. $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ $x \in [0, 1]$

$$y = 2x^2 + 3x - 2 = (x+2)(2x-1)$$

$$f(x) = |(x+2)(2x-1)| + \frac{1}{2} \sin(2x)$$

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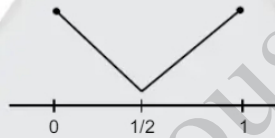
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C-1 when $0 \leq x \leq \frac{1}{2}$
 $f(x) = -(2x^2 + 3x - 2) + \frac{1}{2} \sin(2x)$
 $f(x) = -(4x + 3) + \cos(2x)$
 $x \in [0, \frac{1}{2}] \Rightarrow 4x + 3 \in [3, 5] \Rightarrow -(4x + 3) \in [-5, -3]$
 $f(x) < 0 \forall x \in [0, \frac{1}{2}] \Rightarrow f(x) \downarrow$
 $f(x)_{\max} = f(0) = 2, f(x)_{\min} = f(1/2) = \frac{\sin 1}{2}$

C-2 When $x \in [\frac{1}{2}, 1]$
 $f(x) = (2x^2 + 3x - 2) + \frac{1}{2} \sin(2x)$
 $f(x) = 4(x + 3) + \cos(2x)$
 For $x \in [\frac{1}{2}, 1] \Rightarrow 4x + 3 \in [5, 7]$
 $f(x) > 0 \forall x \in [\frac{1}{2}, 1] \Rightarrow f(x) \uparrow$
 $f(0) = 2, f(\frac{1}{2}) \sin(1), f(1) = 3 + \frac{1}{2} \sin(2)$
 sum of maximum and minimum
 $= 3 + \frac{1}{2}(\sin 2 + \sin 1)$



9. If $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3$ then range of k is
 (1) $[\frac{1}{16}, \frac{5}{8}]$ (2) $[\frac{1}{32}, \frac{7}{8}]$ (3) $[\frac{1}{8}, \frac{9}{8}]$ (4) $[\frac{1}{32}, \frac{5}{8}]$

Ans. (2)

Sol. Let $\tan^{-1}x = t, \cot^{-1}x = \frac{\pi}{2} - t$
 $y = t^3 + \left(\frac{\pi}{2} - t\right)^3$
 $y = t^3 + \left(\frac{\pi^3}{8} - t^3 + \frac{3\pi}{2}t^2 - \frac{3\pi^2}{4}t\right)$

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$$y = \frac{3\pi}{2}t^2 - \frac{3\pi^2}{4}t + \frac{\pi^3}{8}, t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$t = -\frac{b}{2a} = \frac{\frac{3\pi^2}{4}}{2 \times \frac{3\pi}{2}} = \frac{\pi}{4}$$



Range of $y \in \left[F\left(\frac{\pi}{4}\right), F\left(-\frac{\pi}{2}\right) \right]$

$$t = \frac{\pi}{4}, y = \frac{\pi^3}{64} + \frac{\pi^3}{64} = \frac{\pi^3}{32}$$

$$t = -\frac{\pi}{2}, y = -\frac{\pi^3}{8} + \pi^3 = \frac{7\pi^3}{8}$$

$$y \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8} \right] \Rightarrow k \in \left[\frac{1}{32}, \frac{7}{8} \right]$$

10. 3^{2022} divided by 5 then find remainder.

- (1) 4 (2) 3 (3) 2 (4) 1

Ans. (1)

Sol. $9^{1011} = (10 - 1)^{1011} = 10\lambda - 1 = 5\mu - 1$
 \Rightarrow remainder = 4

11. $S = \{ \sqrt{n}, 1 \leq n \leq 50 \}$ n is an odd integer $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ a & 0 & 1 \end{bmatrix}$ $\sum \det(\text{Adj}A) = 100\lambda$ then the value

of λ is
 Ans. (196)

Sol. $|\text{adj}A| = |A|^{n-1}$ n \rightarrow order of det

$$|\text{adj}A| = |A|^2$$

$$|A| = \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 1(1-0) + a(-a) = 1 - a^2$$

$$|\text{adj}A| = (1 - a^2)^2 = (a^2 - 1)^2$$

Now $S = \{ \sqrt{n}, 1 \leq n \leq 50 \}$, n = odd integer

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$$\begin{aligned} \sum \det(\text{Adj}A) &= \sum (a^2 - 1)^2 = \sum (n-1)^2 \\ &= 0^2 + 2^2 + 4^2 + \dots + 48^2 \\ &= 2^2 (1^2 + 2^2 + \dots + 24^2) \\ &= 2^2 \frac{24 \times 25 \times 49}{6} \\ &= 4 \times 4 \times 25 \times 49 \\ &= 100 \times 196 \\ \Rightarrow \lambda &= 196 \end{aligned}$$

12. Let $\{a_i\}_{i=1}^n$ is an AP with common difference 1 where n is an even integer and $\sum_{i=1}^n a_i = 192$ and $\sum_{i=1}^{n/2} a_{2i} = 120$, then find the value of n

Ans. (96)

Sol. $a_1 + a_2 + a_3 + a_4 + \dots + a_n = 192$

$$a_2 + a_4 + a_6 + a_8 + \dots = 120 \dots (i)$$

$$\Rightarrow a_1 + a_3 + a_5 + \dots = 192 - 120 = 72 \dots (ii)$$

By (i) and (2)

$$(a_2 - a_1) + (a_4 - a_3) + \dots = 48$$

$$1 + 1 + \dots + \frac{n}{2} \text{ terms} = 48$$



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