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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 24 June, 2022 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1 If α and β are roots of the equation $3x^2 + \lambda x - 1 = 0$ such that $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$ then value of $6(\alpha^3 + \beta^3)^2$ is

(1) 22 (2) 24 (3) 27 (4) 30

Ans. (2)

Sol. $3x^2 + \lambda x - 1 = 0$

$\alpha + \beta = \frac{-\lambda}{3}$ $\alpha\beta = -\frac{1}{3}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$ $\Rightarrow \alpha^2 + \beta^2 = 15\alpha^2\beta^2$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 15(\alpha\beta)^2$

$\Rightarrow \frac{\lambda^2}{9} - 2\left(-\frac{1}{3}\right) = 15\left(-\frac{1}{3}\right)^2$

$\Rightarrow \frac{\lambda^2}{9} + \frac{2}{3} = \frac{5}{3} \Rightarrow \lambda^2 = 9$

$6(\alpha^3 + \beta^3)^2 = 6\left((\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)\right)^2$

$= 6\left(-\frac{\lambda^3}{27} + \left(-\frac{\lambda}{3}\right)\right)^2$

$= 6\frac{\lambda^2}{9}\left(\frac{\lambda^2}{9} + 1\right)^2$

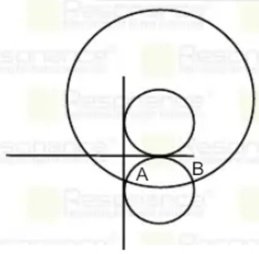
$= 6 \times \frac{9}{9} \times \left(\frac{9}{9} + 1\right)^2 = 6 \times 4 = 24$

2. If $S = \{z \in c : 1 \le |z - (1 + i)| \le 2\}$ and $A = \{z \in s : |z - (1 - i)| = 1\}$ then A is

(1) null set
(2) singleton set
(3) exactly two element
(4) infinite elements

Ans. **(4)**

Sol.



Set A contains all points on minor are AB of circle $|z - (1 - i)| = 1$

3. Let S be the solution set of $\sin\theta \tan\theta + \tan\theta = \sin^2\theta$.

Such that $\theta \in (-\pi, \pi)$, $\theta \ne \pm\frac{\pi}{2}$ and $t = \sum\cos 2\theta$, where θ is solution of above equation then value of $t + n(S)$ is.

(1) 8
(2) 5
(3) 6
(4) 9

Ans. **(2)**

Sol. $\sin\theta \tan\theta + \tan\theta = \sin 2\theta$

$\tan\theta(\sin\theta + 1) - 2\sin\theta\cos\theta = 0$

$\sin\theta\left(\frac{(\sin\theta + 1)}{\cos\theta} - 2\cos\theta\right) = 0$

$\sin\theta = 0$ or $\sin\theta + 1 - 2\cos^2\theta = 0$

$\theta = 0$ $\quad \sin\theta + 1 - 2(1 - \sin^2\theta) = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta = -1, \frac{1}{2}$

$\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, $S = \{0, \frac{\pi}{6}, \frac{5\pi}{6}\} \Rightarrow n(5) = 3$

$t = \sum(\theta(2\theta)) = \cos(0) + \cos(\pi/3) + \cos(5\pi/3)$

$= 1 + \frac{1}{2} + \frac{1}{2} = 2$

$t + n(5) = 2 + 3 = 5$

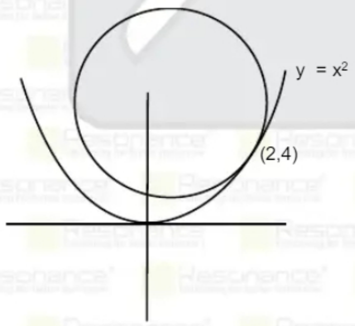
t + n (5) = 2+ 3 = 5

4. A circle of equation $x^2 + y^2 + Ax + By + C = 0$ passes through (0.6) and touches the parabola $y = x^2$ at (2,4) then A + C is

(1) 16 (2) 25 (3) 32 (4) 4

Ans. **(1)**

Sol.



Equation of tangent to parabola at (2,4) $y - 4 = m(x - 2)$

$\frac{dy}{dx} = 2x \Rightarrow m = \left.\frac{dy}{dx}\right|_{(2,4)} = 4$

$y - 4 = 4(x - 2) \Rightarrow 4x - y - 4 = 0$

Let equation of circle be $S + \lambda L = 0$

$(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

It passes through (0,6) $\Rightarrow 4 + 4 + \lambda(0 - 6 - 4) = 0$

$\lambda = \frac{8}{10} = \frac{4}{5}$

$(x - 2)^2 + (y - 4)^2 + \frac{4}{5}(4x - y - 4) = 0$

$x^2 + y^2 + \left(\frac{16}{5}x - 4x\right) + \left(-8y - \frac{4}{5}y\right) + \left(20 - \frac{16}{5}\right) = 0$

$x^2 + y^2 - \frac{4}{5}x - \frac{44y}{5} + \frac{84}{5} = 0$

$A = -\frac{4}{5}, B = -\frac{44}{5}, C = \frac{84}{5}$

$A + C = \frac{80}{5} = 16$

5. If $x(y) = x$ and $y\frac{dx}{dy} = 2x + y^3(y + 1)e^y$ and $x(1) = 0$, then $x(e)$ is equal to

(1) $(e^e - 1)$ (2) $e^3(e^e - 1)$ (3) $e^e - 1$ (4) $e^e - 3$

Ans. **(2)**

Sol. $\frac{dx}{dy} - \frac{2x}{y} = y^2(y+1)e^y$

I.F. = $e^{\int -\frac{2}{y}dx} = e^{-2\ln y} = \frac{1}{y^2}$

$\frac{x}{y^2} = \int (y+1)e^y dy$

$\frac{x}{y^2} = y.e^y + C$

$0 = e + C \Rightarrow C = -e$

$\Rightarrow \frac{x}{y^2} = ye^y - e \Rightarrow \frac{x}{e^2} = e.e^e - e$

$\Rightarrow x = e^3(e^e - 1)$

6. A balloon, spherical in shape is inflated and its surface area is increasing with a constant rate, initially the radius is 3 units, after 5 sec radius is 7 units then the radius after 9 second is

(1) 9 (2) 7 (3) 5 (4) 3

Ans. (1)

Sol. $A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = k$

$4\pi r^2 = kt + C$

at $t = 0, r = 3 \Rightarrow 36\pi = C$

at $t = 5, r = 7 \Rightarrow 4\pi \times 49 = 5k + 36\pi$

$\Rightarrow 5K = 4\pi (49 - 9)$

$5K = 4\pi \times 40$

$K = 32\pi$

$\Rightarrow 4\pi r^2 = 32\pi t + 36\pi$

$\Rightarrow r^2 = 8t + 9 \Rightarrow r^2 = 81 \Rightarrow r = 9$

7. If $\hat{a}$ & $\hat{b}$ are unit vectors and vectors $\vec{c}$ & $\hat{a}$ have an angle of $\frac{\pi}{12}$ between them such that $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$ then the value of $|6\vec{c}|^2$ is.

(1) $3+\sqrt{3}$ (2) $3-\sqrt{3}$ (3) $2+\sqrt{3}$ (4) $2-\sqrt{3}$

Ans. **(1)**

Sol. $|\vec{b}| = \sqrt{c^2 + 4(\vec{c} \times \vec{a})^2 + 0}$

$1 = c^2 + 4c^2 \sin^2 \frac{\pi}{12}$

$1 = c^2 + 4c^2 \times \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

$1 = c^2 + c^2 \left(\frac{4-2\sqrt{3}}{2}\right)$

$1 = c^2 + c^2 \left(2-\sqrt{3}\right)$

$1 = c^2 \left(3-\sqrt{3}\right)$

$3+\sqrt{3} = c^2 \left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)$

$3+\sqrt{3} = 6c^2$

8. If $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ $x \in [0, 1]$, then the sum of absolute maxima and absolute minima is.

(1) $3+\frac{1}{2}(\sin 2+\sin 1)$ (2) $3-\frac{1}{2}(\sin 2+\sin 1)$

(3) $3-\frac{1}{2}(\sin 2-\sin 1)$ (4) $3+\frac{1}{2}(\sin 2-\sin 1)$

Ans **(1)**

Sol. $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ $x \in [0, 1]$

$y = 2x^2 + 3x - 2 = (x + 2)(2x - 1)$

$f(x) = |(x + 2)(2x - 1)| + \frac{1}{2}\sin(2x)$

C-1 when $0 \le x \le \frac{1}{2}$

$f(x) = -(2x^2 + 3x - 2) + \frac{1}{2}\sin(2x)$

$f'(x) = -(4x + 3) + \cos(2x)$

$x \in [0, \frac{1}{2}) \Rightarrow 4x + 3 \in [3, 5) \Rightarrow -(4x + 3) \in [-5, -3]$

$f'(x) < 0 \ \forall\ x \in [0, \frac{1}{2}) \Rightarrow f(x)\downarrow$

$f(x)_{max} = f(0) = 2,\ f(x)_{min} = f(1/2) = \frac{\sin 1}{2}$

C-2 When $x \in \left[\frac{1}{2}, 1\right]$

$f(x) = (2x^2 + 3x - 2) + \frac{1}{2}\sin(2x)$

$f'(x) = 4(x + 3) + \cos(2x)$

For $x \in \left[\frac{1}{2}, 1\right] \Rightarrow 4x + 3 \in [5, 7]$

$f'(x) > 0\ \forall\ x \in \left[\frac{1}{2}, 1\right] \Rightarrow f(x)\uparrow$

$f(0) = 2,\ f\left(\frac{1}{2}\right)\sin(1),\ f(1) = 3 + \frac{1}{2}\sin(2)$

sum of maximum and minimum

$= 3 + \frac{1}{2}(\sin 2 + \sin 1)$

9. If $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3$ then range of k is

(1) $\left[\frac{1}{16}, \frac{5}{8}\right)$ (2) $\left[\frac{1}{32}, \frac{7}{8}\right)$ (3) $\left[\frac{1}{8}, \frac{9}{8}\right)$ (4) $\left[\frac{1}{32}, \frac{5}{8}\right)$

Ans. (2)

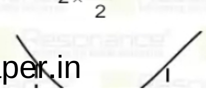
Sol. Let $\tan^{-1}x = t,\ \cot^{-1}x = \frac{\pi}{2} - t$

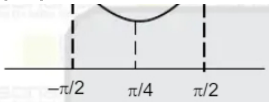
$y = t^3 + \left(\frac{\pi}{2} - t\right)^3$

$y = t^3 + \left(\frac{\pi^3}{8} - t^3 + \frac{3\pi}{2}t^2 - \frac{3\pi^2}{4}t\right)$

$y = \frac{3\pi}{2}t^2 - \frac{3\pi^2}{4}t + \frac{\pi^3}{8},\ t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$t = -\frac{b}{2a} = \frac{\frac{3\pi^2}{4}}{2 \times \frac{3\pi}{2}} = \frac{\pi}{4}$





Range of y $\in \left[F\left(\frac{\pi}{4}\right), F\left(-\frac{\pi}{2}\right)\right]$

$t = \frac{\pi}{4}$, $y = \frac{\pi^3}{64} + \frac{\pi^3}{64} = \frac{\pi^3}{32}$

$t = -\frac{\pi}{2}$, $y = -\frac{\pi^3}{8} + \pi^3 = \frac{7}{8}\pi^3$

$y \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right] \Rightarrow k \in \left[\frac{1}{32}, \frac{7}{8}\right)$

10. 3^{2022} divided by 5 then find remainder.

(1) 4 (2) 3 (3) 2 (4) 1

Ans. **(1)**

Sol. $9^{1011} = (10 - 1)^{1011} = 10\lambda - 1 = 5\mu - 1$

$\Rightarrow$ remainder = 4

11. $S = \{\sqrt{n}, 1 \le n \le 50\}$ n is an odd integer $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ a & 0 & 1 \end{bmatrix}$ $\sum \det(\text{Adj}\,A) = 100\lambda$ then the value of λ is

Ans. **(196)**

Sol. $|\text{adj}A| = |A|^{n-1}$ n $\rightarrow$ order of det

$|\text{adj}A| = |A|^2$

$|A| = \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 1(1 - 0) + a(-a) = 1 - a^2$

$|\text{adj}A| = (1 - a^2)^2 = (a^2 - 1)^2$

Now $S = \{\sqrt{n}, 1 \le n \le 50\}$, n = odd integer

$\sum \det(\text{Adj}A) = \sum (a^2 - 1)^2 = \sum (n - 1)^2$

$= 0^2 + 2^2 + 4^2 + \ldots + 48^2$

$= 2^2 (1^2 + 2^2 + \ldots + 24^2)$

$= 2^2 \frac{24 \times 25 \times 49}{6}$

$= 4 \times 4 \times 25 \times 49$

$= 100 \times 196$

$\Rightarrow \lambda = 196$

12. Let $\{a_i\}_{i=1}^{n}$ is an AP with common difference 1 where n is an even integer and $\sum_{i=1}^{n} a_i = 192$ and $\sum_{i=1}^{n/2} a_{2i} = 120$, then find the value of n

Ans. **(96)**

Sol. $a_1 + a_2 + a_3 + a_4 + \ldots\ldots\ldots + a_n = 192$

$a_2 + a_4 + a_6 + a_8 + \ldots\ldots\ldots + = 120 \ldots..(i)$

$\Rightarrow a_1 + a_3 + a_5 + \ldots\ldots\ldots + = 192 - 120 = 72 \ldots.(ii)$

By (i) and (2)

$(a_2 - a_1) + (a_4 - a_3) + \ldots\ldots = 48$

$1 + 1 + \ldots\ldots \frac{n}{2}$ terms = 48

$\frac{n}{2} = 48 \Rightarrow n = 96$

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